# Week 1 Efficiency in Production and in Consumption and the Fundamental Theorems of Welfare Economics

## 1. Introduction

Much of economic theory of the textbook variety is a celebration of the free market system. This celebration has two parts. First, the operation of the price system, in the context of competitive markets, leads to balance between the demand and supply of the different goods and services traded. In other words, flexible prices result in competitive markets clearing. Second, the market-clearing equilibrium - brought about through flexible prices and competitive markets - is a "good thing" in the sense that it is also a point of economic efficiency<sup>1</sup>. In other words competitive outcomes are also efficient ones. The fact that competition leads to efficiency is known as the *First Fundamental Theorem of Welfare Economics*<sup>2</sup>.

These results - which are, of course, a vindication of Adam Smith's intuition about the existence of an "invisible hand" bringing consistency and order to the chaos of individual actions - would be remarkable in themselves. But there is more. The efficient outcome will have been brought about through parsimony in the use of information; the only things that individuals, in making their supply/demand decisions, need to know are the prices of the different commodities. Furthermore, since the efficient outcome is the result of firms and consumers acting "selfishly", by looking only to their own interests, it is "incentive compatible" in the sense that its existence does not depend upon altruistic behaviour. Lastly, not only will competitive markets lead to an efficient outcome, but any efficient outcome that one might desire can be attained through the operation of competitive markets. This last statement - known as the Second Fundamental Theorem of Welfare Economics - is a very powerful result for it says that if one does not like the particular efficient outcome (perhaps because there were great inequalities associated with it) that resulted from the operation of competitive markets in a specific context, then all is not

<sup>&</sup>lt;sup>1</sup> Economists regard an outcome as being "efficient" if there no other another outcomes in which, relative to the original outcome, some persons are better off without anyone being worse off. If such "better" outcomes exist, then the original outcome is termed inefficient. <sup>2</sup> See Arrow and Hahn (1971), for an authoritative account of how competitive economies work.

lost. In such a situation all that is required is to specify a different, more desirable, outcome and to modify the context suitably; competitive markets operating in the new context would then lead to that outcome.

Against the background of these results the government does not have much of a role. If economic outcomes were not socially desirable then one role for government would be to change the context within which markets operated. This context is provided by the initial endowments with which individuals are equipped for trade in the market. For example, persons who were wealthy or who possessed skills and education would be better equipped for trade than the poor and the unskilled and hence would benefit disproportionately from market outcomes. If endowments were unfairly distributed then market outcomes, notwithstanding the fact that they were efficient, would also be unfair.

Thus, within the framework of market sovereignty, *redistribution* - whereby initial endowments were altered in order to prevent grossly inequitable outcomes would be an acceptable role for government. Even here, its role would be limited by the injunction that, in the pursuit of redistributive objectives, the government should not, by distorting incentives, pervert the free functioning of markets. Since this injunction could only be satisfied through the highly infeasible instrument of "lump-sum" taxes and transfers (that is, all "rich" persons pay, for example, \$100 each, and all "poor" persons receive the same, irrespective of their wealth or poverty), in practice the redistributive role for government would be non-existent.

Although proponents of free markets concede that government might legitimately have a say, however circumscribed, in the sphere of distribution (that is, in terms of who receives how much) it would deny government any role, other than a purely facilitating one, in the spheres of production and of allocation (that is, in terms of deciding what, how much of, and by whom, commodities are to be produced.

In terms of 'by whom to produce', the basic choice is between production by the private sector and production by the public sector. Economists who believe in

the free functioning of markets would argue that the most useful role that government could play in this regard would be to abdicate its productive responsibilities in favour of the private sector - a process known as *privatisation*. Such economists would, in similar vein, argue that the most useful contribution that governments could make to allocative decisions (relating to what, and how much, to produce) would be to remove *market imperfections*. These imperfections, which prevent markets from functioning properly, are associated with an absence of competition (for example, through the existence of monopolies) or with the presence of barriers to price flexibility (for example, through price-support mechanisms like minimum wage legislation). The task of government would then be to take the necessary steps to ensure that all impediments to the proper functioning of markets were removed.

Economists who are not content with this purely passive role for public policy, point to numerous real-world instances where, notwithstanding the existence of competition and price flexibility, markets fail to deliver on efficiency. (Indeed as Solow (1993) has pointed out, many of the young stars of economics, of the past twenty years, made their mark by going beyond the simple competitive model and considering the consequences of dropping some of its restrictive assumptions). In the presence of such cases of *market failure*, they would argue, governments have no alternative but to intervene actively to help markets overcome these difficulties. Indeed Stiglitz (1989) has argued that , contrary to the traditional view that market failures are the exception, such failures may be so pervasive as to be the norm.

However, it is not at all obvious that government will necessarily succeed where markets have failed. Consequently, not all cases of market failure will be amenable to correction through government action. The key to effective government intervention, therefore, lies not in demonstrating the existence of market failures (and thereby establishing a rationale for government intervention) but rather one of identifying situations where such failures are of the kind that would make intervention worthwhile.

# 2. The Conditions for Efficiency

*Efficiency in production* arises when factors of production are allocated between the production of different commodities in such a way that a different allocation would not produce more of some commodities without also producing less of some other commodities. The necessary and sufficient condition for efficiency in production, when there are two factors of production, labour and capital - whose quantities are represented by *L* and *K*, respectively – used in producing two commodities, whose quantities are represented by *X* and *Y*, respectively, is:

$$MRTS_{LK}^{X} = MRTS_{LK}^{Y}$$
(1)

where: MRTS is *the marginal rate of technical substitution* between labour and capital<sup>3</sup>.

*Efficiency in consumption* arises when the available quantities of the different commodities are distributed between the different consumers in such a way that a different distribution would not lead some consumers to be better off without making some other consumers worse off. The necessary and sufficient condition for efficiency in consumption, when fixed quantities of two commodities, denoted *X* and *Y*, are to be distributed between two consumers, *A* and *B* is:

$$MRS^{A}_{XY} = MRS^{B}_{XY}$$
<sup>(2)</sup>

where: MRS is *the marginal rate of substitution* between the two commodities. labour and capital<sup>4</sup>.

Efficiency in production is illustrated in Figure 1: each point on the *efficiency locus* in Figure 1 represents a point of tangency between isoquants and can be mapped onto a point on the production possibility curve in Figure 2:  $W \rightarrow D$  and  $V \rightarrow C$ . Figure 3 illustrates efficiency in consumption and Figure 4 shows the mapping of the efficient points in consumption (of Figure 3) onto a utility possibility curve.

<sup>&</sup>lt;sup>3</sup> MRTS is the reduction in the amount of capital needed when an additional unit of labour is used in the production of a given amount of output.

<sup>&</sup>lt;sup>4</sup> MRS is the amount of one commodity a consumer is prepared to give up, to get an additional unit of another commodity, utility remaining unchanged.



Figure 3: The Edgeworth Box in Consumption

Each point on the production possibility curve represents an efficient allocation of labour and capital in production; given a point on the production possibility curve – representing the total quantities produced of each of the two commodities - each point on the utility possibility curve represents an efficient distribution of the two commodities in consumption.

Allocative efficiency occurs when the "right" amounts of the two commodities are produced or, in other words, the economy is at the "right" point on its production possibility curve. The necessary and sufficient condition for allocative efficiency is:

$$MRS_{XY}^{A} = MRS_{XY}^{B} = MRT_{XY}$$
(3)

where:  $MRT_{XY}$  is the marginal rate of transformation between the two commodities<sup>5</sup>. The point G in Figure 5, below, is a point of allocative efficiency: at G, the slope of the production possibility curve ( $MRT_{XY}$ ) is equal to the (common) slopes of the indifference curves ( $MRS_{XY}^{A} = MRS_{XY}^{B}$ ).



See the Mathematical Appendix for a derivation of the efficiency conditions of equation (3).

## 3. The Fundamental Theorems of Welfare Economics

The First Theorem: Every Competitive Equilibrium is an Efficient Outcome Under a competitive equilibrium, every consumer faces the same prices. Therefore, equilibrium for consumers A and B occurs when:

 $MRS_{XY}^{A} = p_{X} / p_{Y}$  and  $MRS_{XY}^{B} = p_{X} / p_{Y} \implies MRS_{XY}^{A} = MRS_{XY}^{B}$ .

<sup>&</sup>lt;sup>5</sup> Suppose MRT<sub>XY</sub>= $\alpha$  and MRS<sub>XY</sub>= $\beta$ ,  $\alpha$ > $\beta$ . Then if A loses one unit of X, because one unit less of X is produced, A will need to be compensated by  $\beta$  units of Y; but producing one unit less of X, will increase output of Y by  $\alpha$ > $\beta$ . So, after A has been compensated, following a unit reduction in the production of X, there will be a 'surplus' of Y.

Similarly, each producer faces the same input prices. Therefore, equilibrium for each producer occurs when:  $MRTS_{LK}^{X} = q_{K} / q_{L}$  and  $MRTS_{LK}^{Y} = q_{K} / q_{L}$  $\Rightarrow MRTS_{LK}^{X} = MRTS_{LK}^{Y}$  for  $\forall i, j = 1...N$ 

Lastly,  $MRT_{XY} = MC_X / MC_Y$  and, under competitive conditions:  $p_X = MC_X$  and  $p_Y = MC_Y \implies p_X / p_Y = MRT_{XY} = MRS_{XY}^A = MRS_{XY}^B$  so that the conditions for allocative efficiency are satisfied.

Second Theorem: If preferences and technology are convex, then every Pareto optimal outcome can be represented as a competitive equilibrium for some pattern of initial endowments and some vector of prices.



In Figure 6, consumers with an initial endowment at E, when faced with prices represented by the slope of the dashed line, will arrive at the efficient point, H. Similarly, producers will produce at G. Consequently, the Pareto optimal outcome at G and H can be attained through the competitive process.

#### **Mathematical Appendix**

### **Deriving the Efficiency Conditions**

A Pareto efficient allocation is one that maximises one person's utility (A) subject to the constraint that the second person's utility does not change:

$$\underset{X_{1}^{A}, X_{2}^{A}, X_{1}^{B}, X_{2}^{B}}{Max} U^{A}(X_{1}^{A}, X_{2}^{A}) \text{ s.t. } U^{B}(X_{1}^{B}, X_{2}^{B}) = \overline{U}^{B} \text{ and } T(X_{1}, X_{2}) = 0$$
(4)

The Lagrangian for this problem is:

$$L = W[U^{A}(X_{1}^{A}, X_{2}^{A}), U^{B}(X_{1}^{B}, X_{2}^{B})] - m[T(X_{1}, X_{2}) - 0]$$
(5)

and the first order conditions for maximising the Lagrangian with respect to the choice variables are:

$$\frac{\partial L}{\partial X_1^A} = \frac{\partial U^A}{\partial X_1^A} - \mathbf{m} \frac{\partial T}{\partial X_1} = 0$$

$$\frac{\partial L}{\partial X_2^A} = \frac{\partial U^A}{\partial X_2^A} - \mathbf{m} \frac{\partial T}{\partial X_2} = 0$$

$$\frac{\partial L}{\partial X_1^B} = -\mathbf{l} \frac{\partial U^B}{\partial X_1^B} - \mathbf{m} \frac{\partial T}{\partial X_1} = 0$$

$$\frac{\partial L}{\partial X_2^B} = -\mathbf{l} \frac{\partial U^B}{\partial X_2^B} - \mathbf{m} \frac{\partial T}{\partial X_2} = 0$$
(6)

Dividing the first equation by the second, and the third equation by the fourth, yields:

$$MRS_{12}^{A} = MRS_{12}^{B} = MRT_{12}$$
<sup>(7)</sup>

### **Key Points** Weeks 1 &2

- 1. Efficiency in Production: A reallocation of resources from one use to another cannot cause some outputs to increase without a loss in other outputs
- 2. Inefficiency in Production: A reallocation of resources from one use to another can cause some outputs to increase without a loss in other outputs
- 3. Marginal Rate of Technical Substitution (MRTS): amount of an input one can give up for an unit increase in another input, output remaining unchanged, MRTS  $^{X}_{LK}$
- 4. Condition for efficiency in production: MRTS  $_{LK}^{X}$  MRTS  $_{LK}^{Y}$
- 5. Edgeworth Box: isoquants; efficiency locus (tangency of isoquants); derivation of production possibility curve
- 6. Marginal Rate of Transformation: MRTxv
- 7. Efficiency in Consumption: A redistribution of resources from some consumers to others

cannot cause some utilities to increase without a loss in other others

8. Inefficiency in Consumption: A redistribution of resources from some consumers to others

can cause some utilities to increase without a loss in other others

- 9. Marginal Rate of Substitution (MRS): amount of a good one is willing to give up for an unit increase in another good, utility remaining unchanged,  $MRS^{A}_{XY}$ 10. Condition for efficiency in consumption:  $MRS^{A}_{XY} = MRS^{B}_{XY}$
- 11. Edgeworth Box: indifference curves; efficiency locus (tangency of indifference curves); derivation of utility possibility possibility curve
- 12. Condition for Allocative Efficiency. MRTS  $_{LK}^{X}$  = MRTS  $_{LK}^{Y}$  = MRT $_{XY}$ Under *allocative efficiency* we have:
- Efficiency in production and (i)
- the "right" quantities being produced (ii)
- 15. Why competitive markets lead to Efficient Outcomes