Measuring Inequality

1. Lorenz Curves

We consider a collection of N income units (households, individuals) such that y_i represents the income of the i[th] income unit, these incomes being organised in ascending order:

$$y_1 \leq y_2 \leq \ldots \leq y_N$$

In order to construct a Lorenz curve, the cumulated percentage of the income units are plotted (horizontal axis) against the cumulated percentage of income (vertical axis).

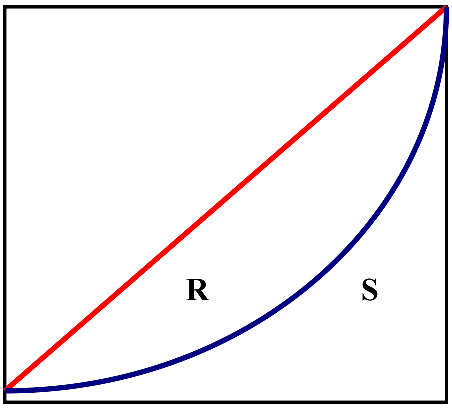

Figure 1: Lorenz Curve

The 45^0 line is the line of perfect equality - along this line, the bottom x% of income units always receive x% of income: x=0…100. In mathematical terms, the Lorenz curve can be represented as:

$$L(x) = L(j/N) = 100 \times \sum_{i=1}^{j} y_i / Y,\ 0 \leq \text{j} \leq \text{N}$$

where: $Y = \sum_{i=1}^{N} y_i$

The Lorenz curve provides a very natural method of comparing two distributions. In Figure 2, which plots the Lorenz curve for two distributions, X and Z, the distribution Z is more unequal than X because the lowest x% of households in Z *always* have a lower share of income than in X. In this case we say that X *Lorenz dominates* Z. Note that Lorenz dominance says nothing about the total amount of money being distributed: Z may have a higher mean income than X even though it is less equally distributed.

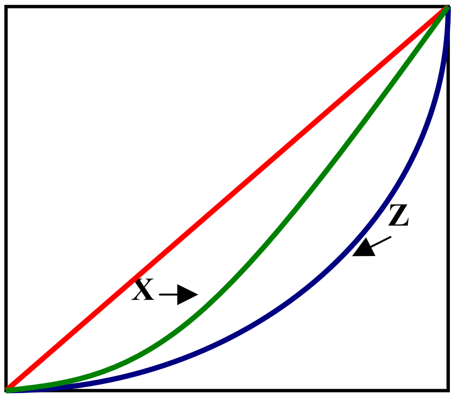

Figure 2: Lorenz Dominance

Figure 3 below shows a case of two Lorenz curves *crossing*. In this case, Z is more equal than X for low levels of income, but X is more equal than Z for high levels of income. Neither distribution Lorenz dominates the other and it is not possible to make an unequivocal statement about which is the more equal distribution. Needless to say, Lorenz curves can cross more than once though, empirically, a single crossing is most common.

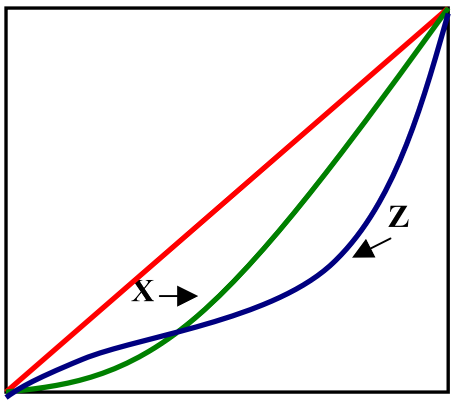

Figure 3: Lorenz Crossing

2. Inequality Measures

An inequality measure, $I(\mathbf{y})$ is a function which assigns a real number to a distribution of income, represented by the vector **y**, such that the value of $I(\mathbf{y})$ increases when, in some meaningful way, inequality can be said to increase.

In constructing an inequality the two *main* properties that it should satisfy are:

(i) ***The Principle of Transfers*** (also known as the ***Pigou-Dalton condition***) whereby a transfer of \$1 from a poorer to a richer person should increase inequality.

(ii) ***Scale Invariance*** whereby an *equi-proportionate* change in all incomes leaves inequality unchanged.

The inequality measures which satisfy the above properties define the class of ***relative inequality mesures***.

It is obvious that some commonly used measures of inequality like the ***range*** ($y_{\max} - y_{\min}$) or the ***relative mean deviation*** ($\frac{1}{2\mu N}\sum_{i=1}^{N}|y_i - \mu|$) violate the Pigou-Dalton condition (μ represents mean income). Other measures, like the ***variance*** ($N^{-1}\sum_{i=1}^{N}(y_i - \mu)^2$) violate scale invariance: a doubling of all incomes will lead to a four-fold increase in the variance. One way of overcoming this problem with the variance is to define the ***coefficient of variation*** as:

$$CV(\mathbf{y};N) = \frac{\sqrt{Variance}}{\mu} \qquad (1)$$

The problem with CV is that the procedure of squaring the deviations from the mean is a very particular one - why use the squaring procedure rather than some other procedure which would also make the inequality measure satisfy the Pigou-Dalton and the scale invariance properties?

Another measure belonging to the class of relative inequality measures is the ***logarithmic variance***:

$$LV(\mathbf{y};N) = N^{-1}\sum_{i=1}^{N}(\log y_i - \log\mu)^2 = N^{-1}\sum_{i=1}^{N}(\log(y_i/\mu))^2 \qquad (2)$$

A third measure belonging to the class of relative inequality measures - and one of the most popular measures of inequality - is the ***Gini coefficient*** $G(\mathbf{y},N)$. In diagrammatic terms, it is defined as:

Area between the Lorenz curve and the diagonal / Total area under the diagonal

$$= \frac{R}{R+S} = \frac{R}{R+(0.5-R)} = 2R = 2(0.5-S) = 1-2S$$

Consequently, $G(\mathbf{y},N) = 0$ when the Lorenz curve coincides with the diagonal (perfect inequality) while $G(\mathbf{y},N) = 1$ when the curve is a right angle (perfect inequality).

Algebraically, the Gini coefficient is represented as:

$$G(\mathbf{y}, N) = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{|y_i - y_j|}{2N^2 \mu} \qquad (3)$$

This shows that the Gini coefficient represents half the mean difference between income pairs divided by mean income: $G(\mathbf{y}, N) = 0.4$ means that for the expected income difference between any two income units chosen at random is 80% of mean income.

One appeal of the Gini coefficient is that it is a very direct measure of income difference, taking account of differences between *every* pair of incomes. As such, it avoids the total concentration on deviations from the mean - and also the arbitrary squaring procedure - of the variance and the coefficient of variation.

Transfer Sensitivity Consider a small transfer of \$δ from a person with \$y to another person with \$(y-x). By the Pigou-Dalton property, all inequality measures belonging to the family of relative inequality measures would record a decrease in inequality. But the amount by which inequality fell would depend on y, the level of income from which the transfer was made. If the inequality measure placed relatively greater weight to transfers at different levels of income - so that the recorded decrease is greater or smaller, depending on the value of y - then it is said to be *transfer sensitive*. The logarithmic variance (equation (2), above)) is transfer sensitive because it places relatively greater weight to transfers at lower, than to higher, levels of income with the consequence that the recorded decrease in the logarithmic variance is greater, the lower the value of y [1]. By contrast, the coefficient of variation (equation (1), above)) is *transfer insensitive* since a transfer of \$δ between two persons, whose incomes were \$z = y-x apart, would cause its value to change by the same amount, irrespective of the income level from which the transfer was being made.

With the Gini coefficient, an income transfer of \$δ from a person with income \$y, to another who is poorer by \$x, reduces the value of the coefficient by more if the two persons are at the *middle* of the distribution than at its ends. This is because transfer sensitivity (of \$δ) with the Gini coefficient depends on the *number of income units* between the incomes (\$y and \$y-x) across which the transfer is effected.

3. Social Welfare and Inequality

The measures of inequality that have been proposed in the literature fall into either of two camps: those which try to measure inequality in some "objective" sense and those which measure inequality in a "normative" sense by establishing a link between changes in inequality and changes in social welfare. In the first camp are the measures discussed above; most prominent in the latter camp is Atkinson's (1970) index, denoted here $AT(\mathbf{y};N)$.

If $u^i(y_i)$ represents the utility of the i[th] individual from his/her income then W, the social welfare function is ***additive*** or ***utilitarian*** if it can be written as the sum of the individual utilities:

$$W(y_1, y_2, .., y_N) = u^1(y_1) + u^2(y_2) + .. + u^N(y_N) \tag{4}$$

From equation (4), social welfare is maximised when the marginal utilities of the different income units are equalised:

$$\frac{du^1}{dy_1} = \frac{du^2}{dy_2} = .. = \frac{du^N}{dy_N} \tag{5}$$

If each person has (is assigned) the same, concave utility function, $u(y_i)$ where $u'(y_i) > 0$ and $u''(y_i) < 0$, then the utilitarian utility function is also egalitarian in the sense that social welfare is maximised when income is equally distributed:

$$y_1 = y_2 = .. = y_N \tag{6}$$

Following from this, define the equally distributed equivalent (ede) income as that level of income, which if equally distributed, would yield the same level of welfare as the existing distribution. If this is represented by y^e, then:

$$W(y^e, y^e, .., y^e) = W(y_1, y_2, .., y_N) \tag{7}$$

The Atkinson index is then defined as:

$$AT(\mathbf{y};N) = 1 - \left(\frac{y^e}{\mu}\right) \tag{8}$$

where: $0 \le AT(y;N) \le 1$. This is illustrated in Figure 4, below.

---

[1] However, it may be so insensitive to transfers among the rich that, for very high incomes, it may violate the Pigou-Dalton condition.

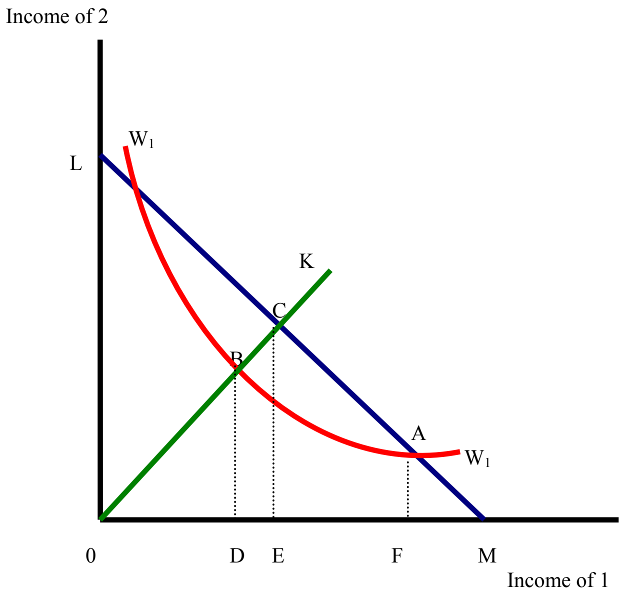

Figure 4
The Equally-Distributed Equivalent Income

In Figure 4, two identical individuals share a given total income OM. The line LM represents all possible distributions of this given total with C as the point of equal division and CE as mean income; W_1W_1 represents a social welfare indifference curve. If the actual distribution of income is represented by the point A, then this is welfare equivalent to each person receiving an income of BD since A and B lie on the same indifference curve. BD is thus the equally distributed equivalent (ede) income corresponding to the actual income distribution at A.

The Atkinson measure of inequality, defined by equation (8), is completely defined by the social welfare function (and, in terms of Figure 4, by the indifference map of the welfare function). Conversely, if one knew the value of the index at every distribution of income, such as A, one could deduce the *ede* and, hence, the social welfare function.

Inequality Aversion The Atkinson index may be interpreted as 1 minus the proportion of mean income that would be needed to maintain, with an equal distribution of income, the existing level of welfare. This would, obviously, depend on how averse one was to inequality and, indeed, a central plank of the Atkinson index is the degree of inequality aversion. In order to compute the Atkinson index, this the degree of inequality aversion needs to be made explicit. From equation (4):

$$dW = \sum_{i=1}^{N} u'(y_i)dy_i = \sum_{i=1}^{N} w_i dy_i$$

where: $w_i = u'(y_i)$ are the welfare weights used when summing the effects of an increase in incomes on changes in welfare.

Suppose that the function $u(y_i)$ takes the specific form:

$$u(y_i) = \frac{y_i^{1-\varepsilon} - 1}{1-\varepsilon} \quad (9)$$

for a parameter ε>0. Then the welfare weights, w_i have constant elasticity (=ε) with respect to y_i since:

$$\frac{\partial w_i}{\partial y_i} = \frac{\partial}{\partial y_i}\left(\frac{\partial u}{\partial y_i}\right) = \frac{\partial y_i^{-\varepsilon}}{\partial y_i} = -\varepsilon y_i^{-\varepsilon-1} \Rightarrow \frac{\partial w_i}{\partial y_i} / \frac{w_i}{y_i} = -\varepsilon y_i^{-\varepsilon-1} / \left(\frac{y_i^{-\varepsilon}}{y_i}\right) = -\varepsilon$$

If a person's income increases then we know that his welfare weight, $w_i = u'(y_i)$, decreases: the question is by how much? The constant elasticity assumption says that the proportionate decrease in the weight, for a given proportionate increase in income, is independent of the level of income: if a person's income increases by 1% - from $100 to $101 or from $100,000 to $101,000 - his welfare weight falls by ε% from its former value. The larger the value of ε, the sharper the fall in the welfare weight for a percentage increase in income. Hence ε is identified as the inequality aversion parameter: the larger the value of ε, the greater the degree of inequality aversion.

Example Suppose a rich person (R) had an income five times that of a poor person (P): $y_R = 5 \times y_P$ and suppose that $1 is to be transferred from R to P. Such a transfer would increase equality (by the Pigou-Dalton condition) and it would also increase welfare (by the concavity of the utility function). But if ε>0, we might be prepared to approve the transfer even if cost R more than $1 to transfer $1 to P. If $\beta \Delta y_P$ (β≥1) represents the amount taken from R to transfer Δy_P to P, without any change in social welfare, then:

$$y_P^{-\varepsilon}\Delta y_P - (5y_P)^{-\varepsilon}(\beta\Delta y_P) = 0 \Rightarrow y_P^{-\varepsilon}\Delta y_P(1-\beta\times 5^{-\varepsilon}) = 0$$
$$\Rightarrow \beta = 5^{\varepsilon} \qquad (10)$$

The table below shows how the maximum sacrifice by the rich person that society would be prepared to tolerate - in order to achieve a $1 increase in the income of the poor person - increases as society's aversion to inequality, as measured by the value of ε>0, increases.

Value of ε	***Amount received by poor person***	***Maximum sacrifice by rich person***
0	**$1**	**$1**
0.5	**$1**	**$1×5$^{0.5}$=2.24**
1.0	**$1**	**$1×5$^{1.0}$=5.00**
1.5	**$1**	**$1×5$^{1.5}$=11.18**
2.0	**$1**	**$1×5$^{2.0}$=25.00**
3.0	**$1**	**$1×5$^{3.0}$=125.00**
4.0	**$1**	**$1×5$^{4.0}$=625.00**

The Atkinson inequality measure (in its operational form) may be obtained by observing that, by definition:

$$\begin{aligned}
&Nu(y^e) = \sum_{i=1}^{N} u(y_i) \\
&\Rightarrow \frac{(y^e)^{1-\varepsilon}}{1-\varepsilon} = N^{-1}\sum_{i=1}^{N}\frac{y_i^{1-\varepsilon}}{1-\varepsilon} \\
&\Rightarrow \left(\frac{y^e}{\mu}\right)^{1-\varepsilon} = N^{-1}\sum_{i=1}^{N}\left(\frac{y_i}{\mu}\right)^{1-\varepsilon} \\
&\Rightarrow \frac{y^e}{\mu} = \left[N^{-1}\sum_{i=1}^{N}\left(\frac{y_i}{\mu}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\
&\Rightarrow AT(\mathbf{y};N) = 1 - \left[N^{-1}\sum_{i=1}^{N}\left(\frac{y_i}{\mu}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}
\end{aligned} \qquad (11)$$

Atkinson's Contributions Atkinson's (1970) classic paper made two important contributions to our understanding of inequality. The first was to provide a method, discussed above, of converting welfare functions into inequality measures. The second was to provide a method for converting inequality measures into welfare functions.

On the second point, the 'Atkinson Theorem' showed that if the Lorenz curves for two income distributions, with the same mean income, do not cross (that is, of two distributions with the same mean income, one distribution Lorenz dominates the other), then any relative inequality index will rank the distributions identically to a ranking obtained using a concave social welfare function.

When mean incomes are different, one needs a specific transformation to link inequality measures to mean income. The reverse Atkinson transformation can be applied to yield[2]:

$$W = \mu(1-I) \tag{12}$$

Equation (12) has the natural interpretation of reducing the welfare emanating from a given level of income by the inequality in its distribution.

4. Inequality and Information Theory

A different motivation for constructing inequality measures is provided by information theory. Suppose that a random variable y can take values $y_1, y_2, ..., y_N$ with probabilities $p_1, p_2, ..., p_N$. The 'information content' of a message that y has taken an unusual value is greater than that of a message that y has taken a more common value. If h_i represents the information content of $y = y_i$ then this is a *decreasing* function of p_i: $h_i = h(p_i),\ h'(p_i) < 0$. In addition, since the values assumed by y are independent of each other, the information content of the joint event, $y = y_i$ and $y = y_j$, is the *sum of the individual information contents*: $h(p_i, p_j) = h(p_i) + h(p_j)$. A function which satisfies these two properties is:

$$h(p_i) = \log(1/p_i) = -\log p_i$$

and the expected amount of information or *entropy* in the system is:

$$E = \sum_{i=1}^{N} p_i h(p_i) = -\sum_{i=1}^{N} p_i \log p_i \tag{13}$$

The maximum value of E occurs when all the events are equally likely so that $p_i = 1/N$ when: $E = \sum_{i=1}^{N}(1/N)h(1/N) = \sum_{i=1}^{N}(1/N)\log(1/N)$. Theil has argued that the entropy concept offers a useful device for measuring inequality by interpreting the N possible outcomes, $y_1, y_2, ..., y_N$, as the incomes of N persons and interpreting the probabilities, $p_1, p_2, ..., p_N$, as their income shares ($y_i / N\mu$). With this interpretation in mind, a measure of inequality is provided by the amount by which *the actual*

---

[2] $W \propto y^e = \mu(1-I)$

amount of entropy falls short of the maximum amount of entropy. This measure, denoted $T(\mathbf{y};N)$ is given by:

$$
\begin{aligned}
T(\mathbf{y};N) &= \sum_{i=1}^{N} \frac{1}{N} h\left(\frac{1}{N}\right) - \sum_{i=1}^{N} p_i h(p_i) \\
&= \sum_{i=1}^{N} p_i \left[h\left(\frac{1}{N}\right) - h(p_i) \right] = \sum_{i=1}^{N} p_i \left[\log p_i - \log\left(\frac{1}{N}\right) \right] \\
&= \frac{1}{N} \sum_{i=1}^{N} \frac{y_i}{\mu} \log\left(\frac{y_i}{\mu}\right)
\end{aligned} \tag{14}
$$

Suppose now that the share of a 'poor' person is increased from p_1 to $p_1 + \Delta p$ with a corresponding reduction in the share of a 'rich' person from p_2 to $p_2 - \Delta p$. Then from equation (14):

$$
\begin{aligned}
& dT = -\sum_{i=1}^{N} \{h(p_i) + p_i h'(p_i)\} \, dp_i \\
& -dp[h(p_1) + p_1 h'(p_1)] + dp[h(p_2) + p_2 h'(p_2)] \\
& [h(p_2) + p_2 h'(p_2) - h(p_1) - p_1 h'(p_1)] \, dp \\
& = [-\log(p_2) - 1 + \log(p_1) + 1] dp \\
& = -\log\left(\frac{p_2}{p_1}\right) dp < 0
\end{aligned} \tag{15}
$$

The size of the reduction in the inequality measure depends *only* on the ratio, p_2 / p_1: a poor-to-rich transfer of income between two persons with income shares of 1% and 2% will lead to the same reduction in the inequality as the same transfer between two persons with income shares of 10% and 20%. Consequently, the shares p_1 and p_2 are the same distance apart as the shares p_3 and p_4 if:

$$
\log\left(\frac{p_2}{p_1}\right) = \log\left(\frac{p_4}{p_3}\right) \Rightarrow h(p_1) - h(p_2) = h(p_3) - h(p_4) \tag{16}
$$

Generalised Entropy The function $h(p_i) = -\log(p_i)$ can be viewed as member of a wider class of funcions:

$$
\begin{aligned}
h(p_i) &= \frac{1 - p_i^{\beta}}{\beta}, \ \beta \neq 0 \\
&= -\log(p_i), \ \beta = 0
\end{aligned} \tag{17}
$$

Deriving an inequality measure in exactly the same manner as before yields the generalised entropy measure:

$$
\begin{aligned}
E(\mathbf{y},N) &= \frac{1}{\beta+\beta^2}\sum_{i=1}^{N} p_i(p_i^{\beta}-N^{-\beta}) \\
&= \frac{1}{\beta(\beta+1)}\left[\sum_{i=1}^{N}\left(\frac{y_i}{N\mu}\right)^{1+\beta} - N^{-\beta}\sum_{i=1}^{N}\left(\frac{y_i}{N\mu}\right)\right] \\
&= \frac{1}{\beta(\beta+1)}\left[N^{-(1+\beta)}\sum_{i=1}^{N}\left(\frac{y_i}{\mu}\right)^{1+\beta} - N^{-\beta}\right]
\end{aligned}
\tag{18}
$$

Because of the presence of the term $N^{-\beta}$ in equation (18), the inequality measure does not satisfy the property of *population homogeneity* (the value of the inequality index should be invariant to replications of the population). To ensure that it does, the expression in equation (18) is multiplied by $N^{-\beta}$ to yield:

$$
\begin{aligned}
E(\mathbf{y};N) &= N^{-\beta}E(\mathbf{y};N) \\
&= \frac{1}{\beta(\beta+1)}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i}{\mu}\right)^{1+\beta} - 1\right] \\
&\frac{1}{\theta(\theta-1)}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i}{\mu}\right)^{\theta} - 1\right]
\end{aligned}
\tag{19}
$$

where: $\theta = 1+\beta$

Two Special Cases: When *β=0* (*θ=1*), the entropy measure is $T(\mathbf{y};N)$ given by equation (14). When *β=-1* (*θ=0*), the entropy measure is the *mean logarithmic deviation index*:

$$
M(\mathbf{y};N) = \frac{1}{N}\sum_{i=1}^{N}\log\left(\frac{\mu}{y_i}\right) \tag{20}
$$

The mean logarithmic deviation index can be represented as the log of the ratio of the arithmetic mean to the geometric mean of incomes. Let $\lambda = \prod_{i=1}^{N}(y_i)^{1/N}$ represent the geometric mean of $y_1, y_{2,} ..., y_N$ and consider the log ratio of the arithmetic mean to the geometric mean:

$$
\Phi = \log\left(\frac{\mu}{\lambda}\right) = \log\mu - \frac{1}{N}\sum_{i=1}^{N}\log y_i = \frac{1}{N}\sum_{i=1}^{N}\log\left(\frac{\mu}{y_i}\right) = M(\mathbf{y};N) \tag{21}
$$

Interpretation of β

The Generalised Entropy family of inequality indices (equation (19), above) is defined by a parameter β (or, equivalently, by a parameter $\theta = 1 + \beta$). All inequality indices should embody the Pigou-Dalton condition (also known as the Principle of Transfers): this says that a transfer from a rich to a poor person must cause the value of the inequality index to fall. But by *how much* the value of the index will fall will depend upon the value of the parameter β.

More formally, from equation (19),

$$E(\mathbf{y}, N) = \frac{1}{\beta + \beta^2} \sum_{i=1}^{N} p_i (p_i^{\beta} - N^{-\beta}) = \frac{1}{\beta + \beta^2} \sum_{i=1}^{N} p_i^{1+\beta} - N^{-\beta}$$

and, therefore, a transfer of dp income share from a richer (*i=2*) to a poorer person (*i=1*) will cause the Generalised Entropy index to fall by:

$$\begin{aligned} dE(\mathbf{y}; N) &= \frac{\beta}{\beta(1+\beta)} (p_1^{\beta} - p_2^{\beta}) dp = \frac{1}{\beta} (p_1^{\beta} - p_2^{\beta}) dp \\ &= \frac{1}{\beta} \left[(1 - p_2^{\beta}) - (1 - p_1^{\beta}) \right] dp = \left[h(p_2) - h(p_1) \right] dp \end{aligned} \qquad (22)$$

Consequently,

$$\frac{\partial}{\partial \beta} \left(\frac{\partial E(\mathbf{y}; N)}{\partial p} \right) = \frac{\partial h(p)}{\partial p_2} - \frac{\partial h(p)}{\partial p_1} = p_1^{\beta-1} - p_2^{\beta-1} < 0 \qquad (23)$$

so that the reduction in inequality, following an egalitarian transfer, falls as the value of the parameter β increases. **So, the value of β may be interpreted as the degree of inequality aversion.**

But β may also be *further* interpreted as the degree of transfer sensitivity. If one defines the *distance* between the income share of the two persons as :

$$\lambda(\beta, p_1, p_2) = h(p_1) - h(p_2) = \frac{p_2^{\beta}}{\beta} - \frac{p_1^{\beta}}{\beta} \geq 0 \qquad (24)$$

then equation (22) shows that the *greater* the distance between two income shares, the *larger* the fall in inequality following a egalitarian transfer.

The ***strong principle of transfers*** says that the reduction in inequality, following an egalitarian transfer should depend *only* on the distance between the shares, *regardless of the parties between which the transfer is made*. So, an egalitarian transfer of Δy between 1 (poor) and 2 (rich) will reduce inequality by the same amount as an egalitarian transfer of Δy between 3 (poor) and 4 (rich) if and only if:

$$\lambda(\beta, p_1, p_2) = \lambda(\beta, p_3, p_4) \tag{25}$$

It is obvious from equation (25) that the family of Generalised Entropy Indices satisfies the strong principle of transfers. Indeed, the *only* class of measures to satisfy the strong principle of transfers is the class of Generalised Entropy Indices.

Now suppose that there are three persons such that: $p_1 < p_2 < p_3$ and $p_2 - p_1 = p_3 - p_2$. Then:

$$\begin{aligned} &\lambda(\beta, p_1, p_2) > \lambda(\beta, p_3, p_2) \text{ if } \beta<1 \\ &\lambda(\beta, p_1, p_2) = \lambda(\beta, p_3, p_2) \text{ if } \beta = 1 \\ &\lambda(\beta, p_1, p_2) < \lambda(\beta, p_3, p_2) \text{ if } \beta > 1 \end{aligned} \tag{26}$$

So, if β<1, the Generalised Entropy index is more sensitive to transfers made at the lower end of the income scale; if β>1, the Generalised Entropy index is more sensitive to transfers made at the upper end of the income scale; and if β=1, the Generalised Entropy index is transfer insensitive. **The parameter β may thus be interpreted as a transfer sensitivity parameter**.

This follows because, from equation (17):

$$\frac{dh(p)}{dp} = -p^{\beta-1} < 0 \text{ and } \frac{d}{dp}\left[\frac{dh(p)}{dp}\right] = -(\beta-1)p^{\beta-2} \tag{27}$$

Consequently, the h(y) curve is: linear if *β=1* ($d^2h/dp^2 = 0$); convex to the origin if *β<1* ($d^2h/dp^2 > 0$); and concave to the origin if *β>1* ($d^2h/dp^2 < 0$).

Both the Theil (*β=0 or θ=1*) and the MLD index (*β=-1 or θ=0*) are sensitive to transfers at the lower end of the scale. When *β=1(θ=2)*, the Entropy index is: $\frac{1}{2}\left(\sum_{i=1}^{N} p_i^2 - \frac{1}{N}\right)$ which is cardinally equivalent to the *Herfindahl index*: $\sum_{i=1}^{N} p_i^2$ and ordnially equivalent to the coefficient of variation which, as is well known, is transfer insensitive.

Example Now suppose that there are three persons such that: $p_1 < p_2 < p_3$ and $p_2 - p_1 = p_3 - p_2$.

	Person	*Income*	*Income Share*
	1	$2,000	0.2%
	2	12,000	1.2%
	3	22,000	2.2%
	Total	1,000,000	100%
β	$\lambda(\beta; p_i, p_j) = h(p_i) - h(p_j)$	Distance between persons 1 and 2	Distance between persons 2 and 3
-1	$\frac{1}{p_i} - \frac{1}{p_j}$	417	378
0	$\log(p_i / p_j)$	179	60.6
1	$p_i - p_j$	0.01	0.01