The Expected Utility Rule for Evaluating Gambles
Suppose that a person is faced with a choice between two gambles: $X = (p_G^X, p_B^X, c_G^X, c_B^X)$ and $Y = (p_G^Y, p_B^Y, c_G^Y, c_B^Y)$, with expected utilities: $EU^X = p_G^X \times c_G^X + p_B^X \times c_B^X$ and $EU^Y = p_G^Y \times c_G^Y + p_B^Y \times c_B^Y$. Then he/she chooses between the gambles by comparing their expected utilities: X is chosen over Y if, and only if, $EU^X > EU^Y$ and Y is chosen over X if and only if $EU^Y > EU^X$ with indifference between X and Y being defined by $EU^Y = EU^X$.

The *certainty equivalent* of a gamble, $X = (p_G^X, p_B^X, c_G^X, c_B^X)$, is defined as the sum of money $CE which, *if received with certainty*, would cause the person to be *indifferent* between accepting and rejecting the gamble: $u(CE) = EU^X$.

The *risk premium* associated with a gamble is the *maximum* amount a person is prepared to pay to avoid a risky situation and is given by: ER-CE. Since, *under risk aversion*, CE<ER , the risk premium associated with a gamble is greater for persons with high risk aversion.

Exercise: Persons A and B, respectively, have utility functions $-e^{-0.00001c}$ and $-e^{-0.00002c}$ and they are each faced with the gambles: X=(0.4: $40000; 0.6: -$10000) and Y=(0.33: $21000; 0.33: $9000; 0.33: -$9000). Compute the CE and risk premium for A and B for each of the gambles X and Y and show that B is more risk-averse than A.

Measuring the Degree of Risk Aversion
Intuitively, the degree of risk aversion is represented by the degree of curvature of the utility function: the more "bowed" the utility function - the greater its curvature - the more risk averse is the person. More formally, the degree of risk aversion of a person whose utility function is *u(c)*, is:

$$\lambda(c) = -\frac{u''(c)}{u'(c)}$$

If $\lambda'(c) < 0$, then risk aversion decreases with increasing wealth; on the other hand, if $\lambda'(c) = 0$, then risk aversion is constant. In the most general case, constant risk aversion is embodied by the functional form:

$$u(c) = -Ae^{-\lambda c} + B$$

where A is a positive constant an B is any constant.

Exercise: Show that the utility functions of A and B, above, embody constant risk aversion.

Exercise (Risk Sharing)**:** Jill, whose utility function is: $u(z) = 12.5859 - 7.4267e^{-0.0000211z}$, is offered a gamble: (0.5: $50,000; 0.5: -$25,000).

(a) Will she take this gamble if her alternative is the certainty of $0?
(b) She prints 100 shares in the gamble, each share representing the gamble: (0.5: $500; 0.5: -$250), and offers each of 100 friends - all of whom have the same utility function as Jill - a share for $100. Will she get any takers on her offer?

Exercise (Insurance): John has assets of \$1 million. Of this amount, \$250,000 is perfectly safe (say, a bank deposit). The remaining \$750,000 is his equity in his house. If the house burns down (with probability=0.05) he loses this amount. He can insure his house against this contingency by paying an insurance company a premium of \$40000. Then if his house does burn down, the insurance company will pay him \$750,000.

(i) What is the expected net earnings to the insurance company from this policy?

(ii) Would John buy this policy if he was *risk neutral*?

(iii) If John's utility function was $u(x) = \sqrt{x}$, where x represents his total assets, would he buy the insurance policy?

(iv) Suppose John can buy partial insurance. This works as follows: John can choose a number α, where: $0 \le a \le 1$. Then, he has to pay a premium $\alpha \times 40,000$ to the insurance company and, if the house burns down, the insurance company will pay out $\alpha \times 750,000$. Under these circumstances, what level of insurance would John select?

Limitations of the Expected Utility Approach

The EU approach to choice uncertainty cannot cope with the *portfolio effect* or with the *temporal resolution of uncertainty effect*.

Portfolio Effect: The value to you of your payoff depends upon the "state of nature" I which it is received.

***Temporal Resolution of Uncertainty*:** The outcome of the same gamble is made known to you at different points in time.

A General Result About Risk Sharing

Suppose that a person (like Jill, in the example above) rejects a gamble because the chance of a loss and/or the size of the loss is significantly high. Not only is the CE<ER (because Jill is risk averse) but in the case of Jill's gamble, her CE<0: he would have to be paid to assume the risk!

Theorem: For any number $\beta<1$, there exists a number $0<\alpha<1$ such that an individual, no matter how risk averse, will accept an α share of the gamble by paying $\alpha\times\beta$ of the expected return (ER) from the gamble.

The more risk averse the potential investors, the riskier the investment, the greater the value of β, *the lower will be value of* α *and the larger will be the number of investors*.

So market failure through risk aversion can be defeated through spreading risks. This usually requires appropriate institutions and markets (insurance, futures).

***Application to Insurance*:** In the insurance example, above, the insurance company is prepared to accept the risk of paying out \$750,000 for a premium of \$40,000. The individual (John) is prepared to pay this because, even after paying \$40,000, the certainty wealth offered by the insurance exceeds the CE of the gamble to the individual. The insurance company is less risk averse than the individual because, as a company, it is able to spread its risk among its shareholders.

Exercise on Risk Sharing:

(i) Jill, in the example above, decides to retain a proportion θ of the gamble and to *give* the rest to other investors. By doing so, the gamble she faces is: (0.5: \$50,000θ; 0.5: -\$25,000θ). What is the optimal value of θ?

(ii) Suppose now that Jill has an associate (with the same utility function as Jill's) to whom she offers 10% of the gamble (i.e. 0.5: \$5,000; 0.5: -\$2,500) at a price of 95% of 10% of the full ER of the gamble (i.e. 0.95×0.10×12,500=\$1,187.50). Will the associate accept this offer?

(iii) Suppose Jill is determined to obtain 95% of the full ER of he gamble. What is the largest share α that Jill can sell her associate for a price of 0.95×12,500α?

(iv) Jill has another associate whose utility function is: $u(z) = 12.5859 - 7.4267e^{-0.00001z}$. Is this associate more/less risk averse than the first associate?

(v) Jill is optimistic about the venture and thinks there is a 70% chance of getting \$50,000 and a 30% chance of losing \$25,000. However, the market does not share her optimism and thinks the chances of these outcomes are 50-50. She decides to retain a proportion θ of the venture and to sell the remainder 1-θ to other investors at a price \$123 per percent. Why does she choose \$123? What is the optimal value of θ?

(vi) Jill is a good saleswoman and she "talks up" the market into believing that her chances of success/ failure are 60-40. What price would she now charge for her venture and how much would she retain?

Exercise on Insurance and Hidden Information

All homeowners in an area own homes worth \$80,000 and an insurance company, Firelite, offers then protection again fire. Firelite offers two different policies: (i) *partial insurance*, whereby a homeowner pays \$5,900 in premium and in the event of a fire receives \$58,400 from Firelite; (ii) *full insurance*, whereby a homeowner pays \$11,600 in premium and in the event of a fire receives \$80,000 from Firelite.

The chance of a fire in a particular home is $0 \le p \le 0.4$ and is known to the homeowner *but not to Firelite*.

Peter and John are two home owners. Peter lives on the edge of a forest and, for him, p=0.1. John lives in a suburb and , for him, p=0.03. Both have the same utility function: $u(x) = \sqrt{10,000 + x}$ where x is the net wealth of the home owner in a particular contingency and under a particular policy: with partial insurance and no fire x=80000-5900 while with partial insurance and a fire x=58400-5900.

Of the three options available - no insurance, partial insurance, full insurance - what is the best option for Peter? For John?

For which values of p (the probability of house burning down) would home owners with that p choose: No insurance? Partial insurance? Full insurance?