Some Calculus

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Functions

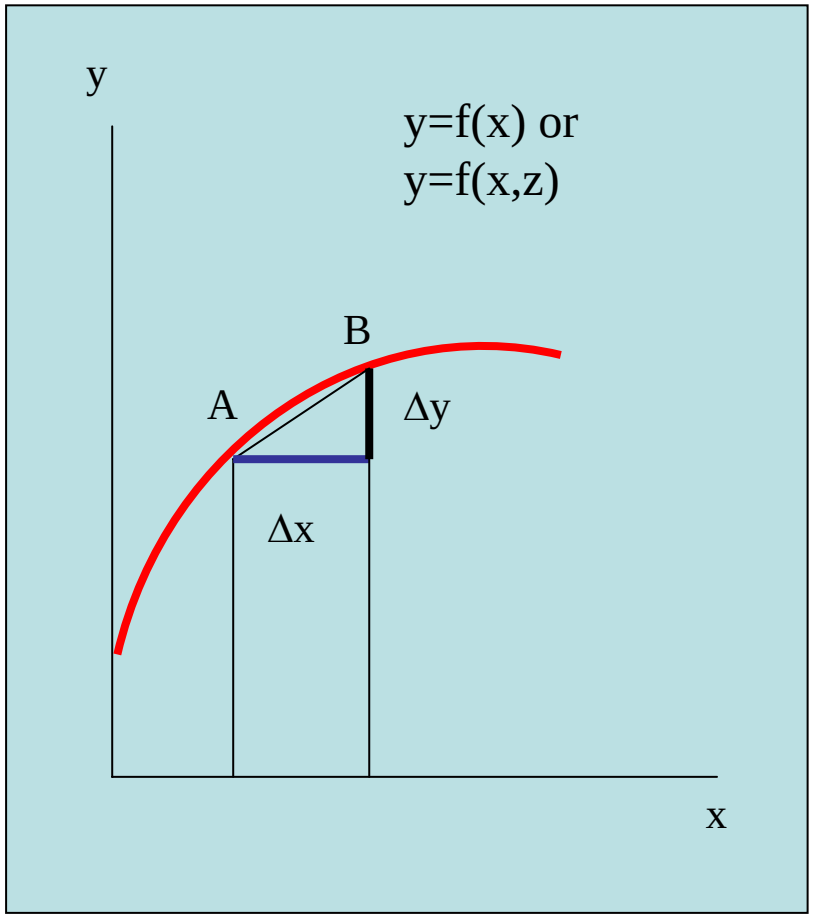
- $y=f(x)$
- $y=f(x,z)$
- $y=f(x_1,x_2 \ldots x_n)$

Q1: dy/dx or df/dx or $f'(x)$

Q2: $\partial y/\partial x$ or $\partial f/\partial x$ or $f_x(x)$

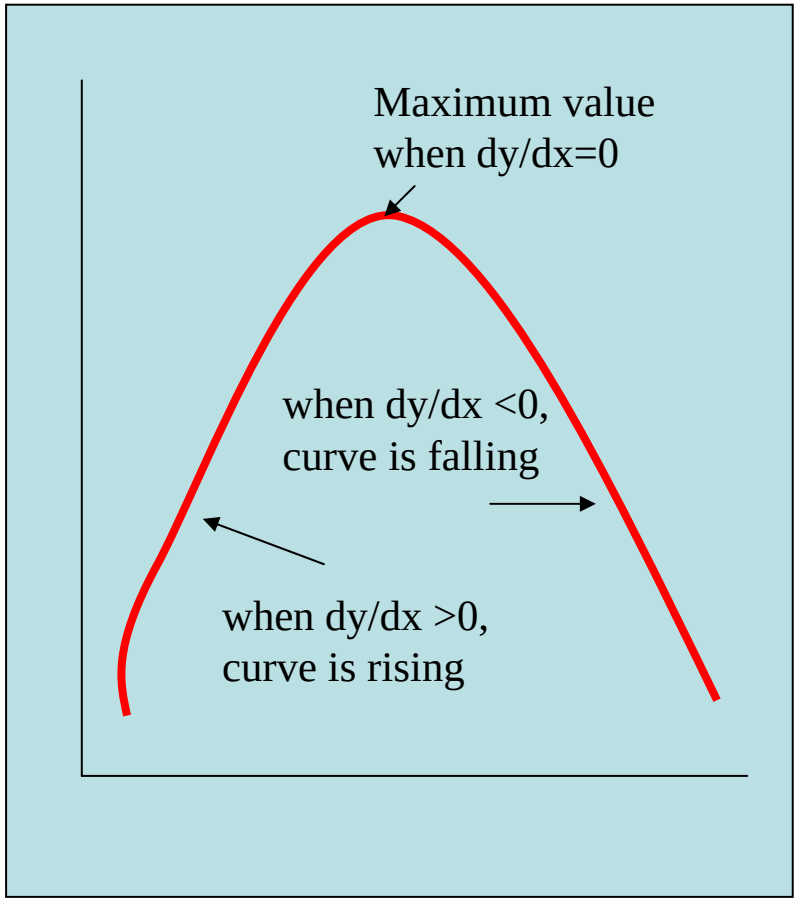
- By how much does the value of y change for a small change in the value of x
- By how much does the value of y change for a small change in the value of x, ***the value of z held constant at some level***

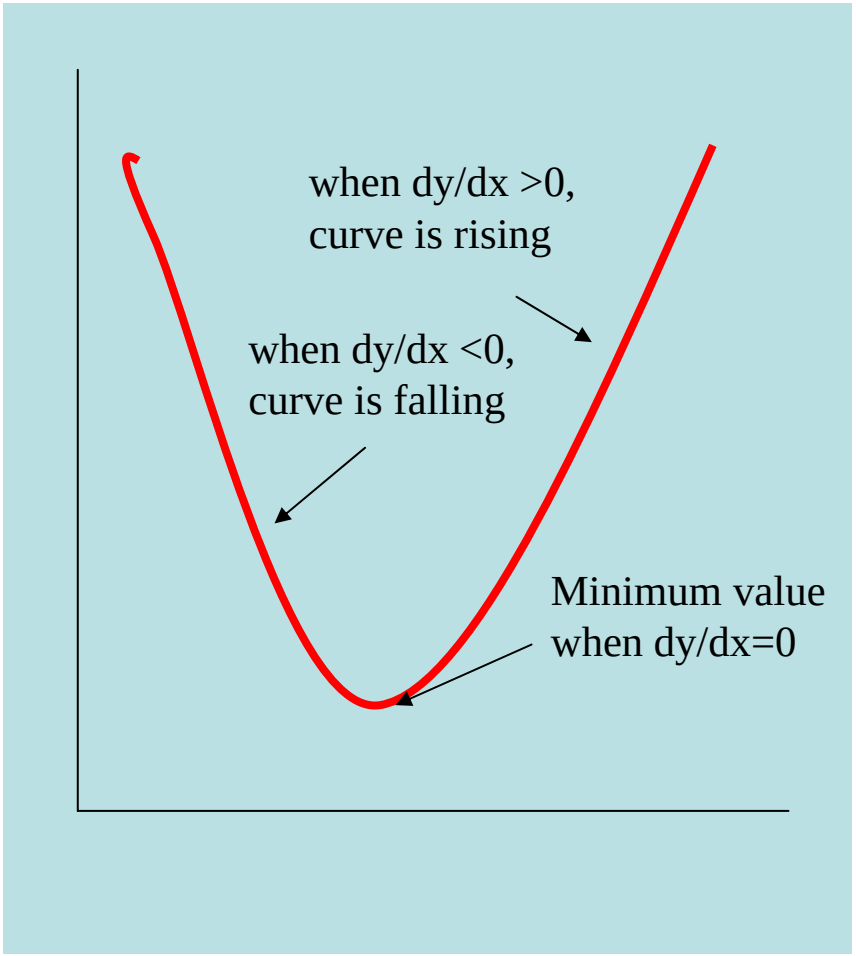
A Picture



- The slope of line AB is $\Delta y/\Delta x$
- As $\Delta x \rightarrow \infty$ $\Delta y/\Delta x \rightarrow dy/dx$ or $\partial y/\partial x$

Another picture





Rule 1

- $y=x^N \Rightarrow dy/dx=Nx^{N-1}$
- Example:

$$y = \sqrt{x} = x^{1/2} \Rightarrow dy/dx = \frac{1}{2} x^{(1/2)-1}$$

$$= \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

Rule 2: multiplication by a constant rule

- If θ is a constant and $y=\theta f(x)$, $dy/dx=\theta f(x)$
- Example:
- $y=\theta x^{1/2}$ then $dy/dx=\theta d(x^{1/2})/dx=\theta \times 0.5 \times (1/\sqrt{x})$

Rule 3: function of a function

- y=f(x) but x=g(z) so y=f(g(z))
- dy/dz = $\frac{dy}{dx}\frac{dx}{dz}$

Rule 4: addition rule

- y=f(x)+g(x) $\frac{dy}{dx} = \frac{df}{dx} + \frac{df}{dy}$

Rule 5: multiplication rule

- $y = f(x) \times g(x)$ $\frac{dy}{dx} = f(x)\frac{dg(x)}{dx} + g(x)\frac{df(x)}{dx}$

Rule 6: division rule

- y=f(x)/g(x)

$$\frac{dy}{dx} = \frac{g(x)\times(df/dx) - f(x)\times(dg/dx)}{[g(x)]^2}$$

Our function

Expected utility has be maximised by choosing appropriate α

$$EU = 0.95 \times \left(1000000 - 40000\alpha\right)^{1/2}$$

$$+0.05 \times \left(250000 + 710000\alpha\right)^{1/2}$$

Parts

First part:

$$\frac{dEU}{d\alpha} = 0.95 \times 0.5 \times \frac{1}{[1000000 - 40000\alpha]^{1/2}} \times (-40000)$$

Second Part

$$\frac{dEU}{d\alpha} = 0.05 \times 0.5 \times \frac{1}{[250000 + 710000\alpha]^{1/2}} \times 710000$$

Parts

$$\frac{dEU}{d\alpha} = -0.95 \times 0.5 \times \frac{1}{[1000000 - 40000\alpha]^{1/2}} \times 40000$$

$$+0.05 \times 0.5 \times \frac{1}{[250000 + 710000\alpha]^{1/2}} \times 710000 = 0$$

Solution

$856500\alpha = 712500$ $\alpha = 0.83$