Risk and Uncertainty

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Basic Concepts

- ***Gamble:*** An action with more than one possible outcome, such that with each outcome there is an associated probability of that outcome occurring. If the outcomes are good (G) and bad (B), denote the associated probabilities by p_G and p_B
- ***Payoff:*** With each outcome is associated a "pay-off" which can be expressed in terms of money: $\$c_G$ and $\$c_B$.
- ***Utility from a Payoff:*** With each payoff is associated a "utility", u(c): $u(c_G)$ is the utility in the good situation $u(c_B)$ is the utility in the bad situation. We assume that: $u'(c)=du/dc > 0$

Expected Return and Utility

- ***Expected Return:*** The expected return from the gamble is: $ER=p_G \times c_G + p_B \times c_B$
- ***Expected Utility:*** The expected utility from the gamble is: $EU=p_G \times u(c_G) + p_B \times u(c_B)$

Expected Utility Rule

- You have a wealth of $W and you are faced with a gamble
- You have to decide whether or not to accept the gamble.
- If you accept the gamble, your expected utility is $EU=p_G \times u(c_G)+p_B \times u(c_B)$
- If you reject the gamble, your certain utility is u(W)
- You compare EU and u(W): accept if EU>u(W); reject if EU<u(W); indifferent if EU=u(W)

A Fair Gamble

- ***Fair Gamble:*** A fair gamble is one in which the sum that is bet (*W*) is equal to the expected return: $W = ER = p_G \times c_G + p_B \times c_B$
- Most gambles are “unfair” – W > ER – that is how casinos stay in business

An Example

- You have an initial wealth of W=$500. You are offered a gamble:
- $250 with $p_B = 0.5$ or $750 with $p_G = 0.5$; ER=$500
- You can accept the gamble or you can decline the gamble
- If you decline you keep $500 with certainty: u(500); If you accept: EU= $0.5 \times u(250) + 0.5 \times u(750)$
- This gamble is called a *fair gamble* because the amount that is bet ($500) is equal to the expected return from the gamble ($500).
- You will *reject* the gamble if u(500)>EU; You will *be indifferent* to the gamble if u(500)=EU; You will *accept* the gamble if u(500)<EU

Another Example

- You have an initial wealth of W=$500. You are offered a gamble:
- $250 with $p_B = 0.6$ or $750 with $p_G = 0.4$; ER=$450
- You can accept the gamble or you can decline the gamble
- If you decline you keep $500 with certainty: u(500); If you accept: EU= $0.5 \times u(250) + 0.5 \times u(750)$
- This gamble is an *unfair gamble* because the amount that is bet ($500) is less than expected return from the gamble ($450).
- You will *reject* the gamble if u(500)>EU; You will *be indifferent* to the gamble if u(500)=EU; You will *accept* the gamble if u(500)<EU

Attitudes to Risk

- Intuitively, whether someone accepts a gamble or not depends on his ***attitude to risk***
- Again intuitively, we would accept "adventurous" persons to accept gambles that more "cautious" persons would reject
- To make these concepts more precise we define three broad attitudes to risk

Three Attitudes to Risk

- The Risk Averse Person
- The Risk Neutral Person
- The Risk Loving Person
- To define these attitudes, we use the concept of a ***fair gamble***
- In essence, a fair gamble allows you receive the ***same*** amount of money through ***two distinct*** ways:
- Gambling or not gambling

Attitudes to Risk and Fair Gambles

- A risk averse person will ***never*** accept a fair gamble
- A risk loving person will ***always*** accept a fair gamble
- A risk neutral person will be ***indifferent*** towards a fair gamble

What Does This Mean?

- Given the choice between earning the ***same*** amount of money through a ***gamble*** or through ***certainty***

❖The risk averse person will opt for certainty

❖The risk loving person will opt for the gamble

❖The risk neutral person will be indifferent

Diminishing Marginal Utility

- Why does the risk averse person reject the fair gamble?
- Answer: because her marginal utility of money diminishes

Example

- Your wealth is $10. I toss a coin and offer you $1 if it is heads and take $1 from you if it is tails
- This is a fair gamble: $0.5\times11+0.5\times9=10$, ***but you reject it***
- Because, your gain in utility from another $1 is ***less*** than your loss in utility from losing $1
- Your MU diminishes, you are risk averse
- Conversely, if you are risk averse, your MU diminishes

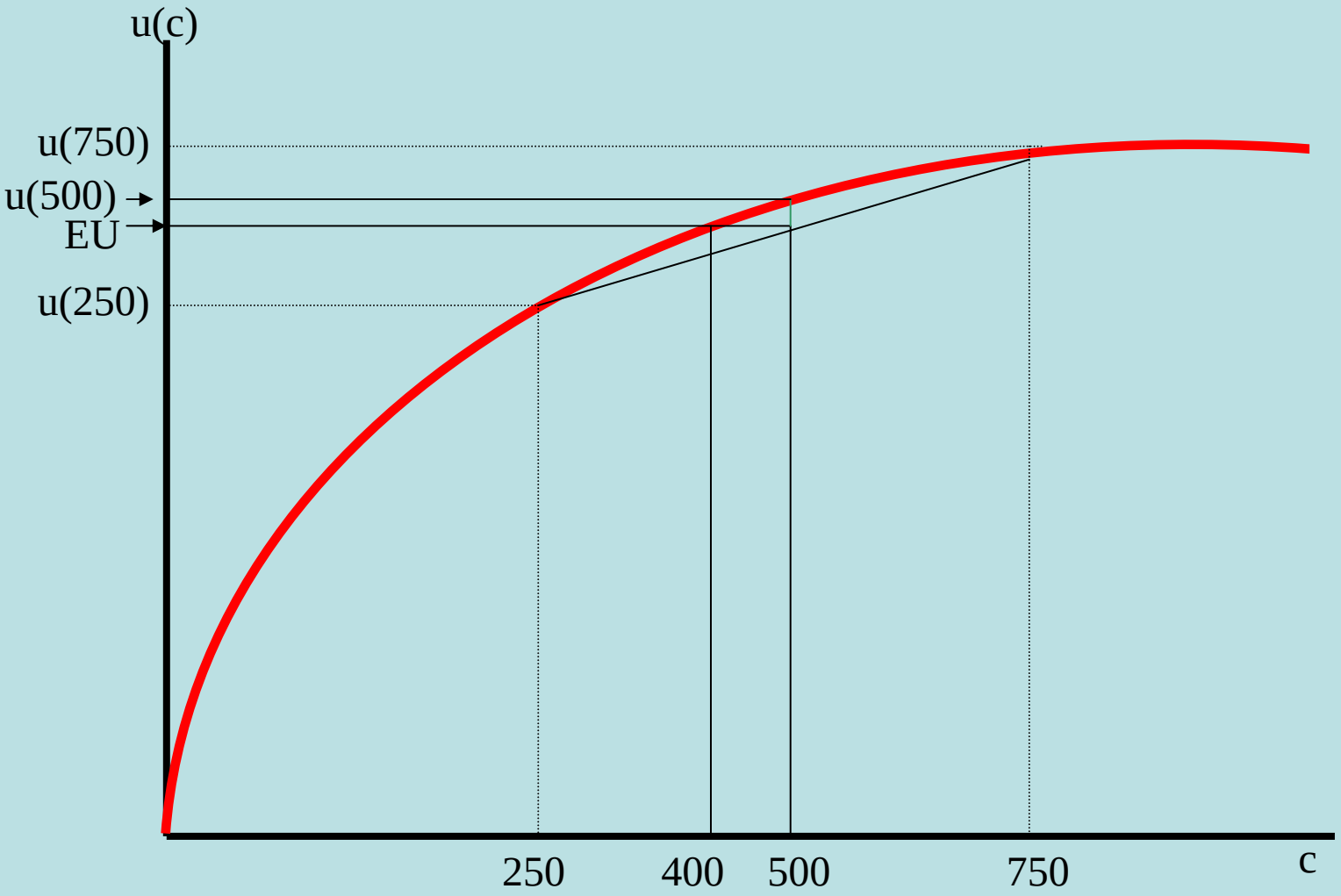
Equivalent Concepts

- A person is risk averse
- A person's marginal utility of money diminishes
- A person's utility function, u(c), is ***concave***

Two Concepts

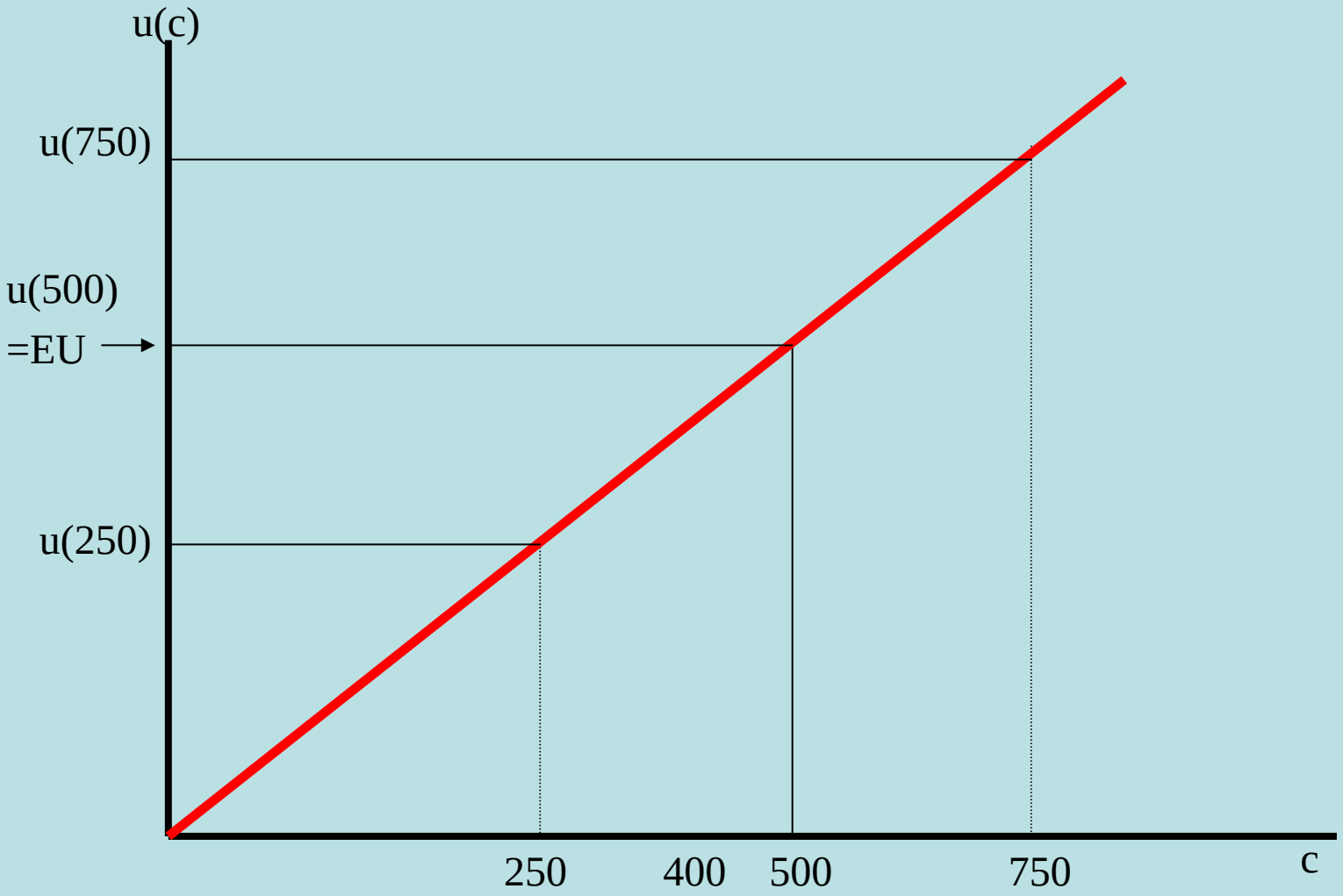
- The ***certainty equivalent*** of a gamble: the sum of money, X, which, if received with certainty will yield the same utility as the gamble
- X is CE if $u(X) = EU = p_G \times u(c_G) + p_B \times u(c_B)$
- The ***risk premium*** associated with a gamble is the maximum amount a person is prepared to pay to avoid the gamble
- RP = ER - CE

A risk averse person / with diminishing MU / with a concave utility function will reject a fair gamble



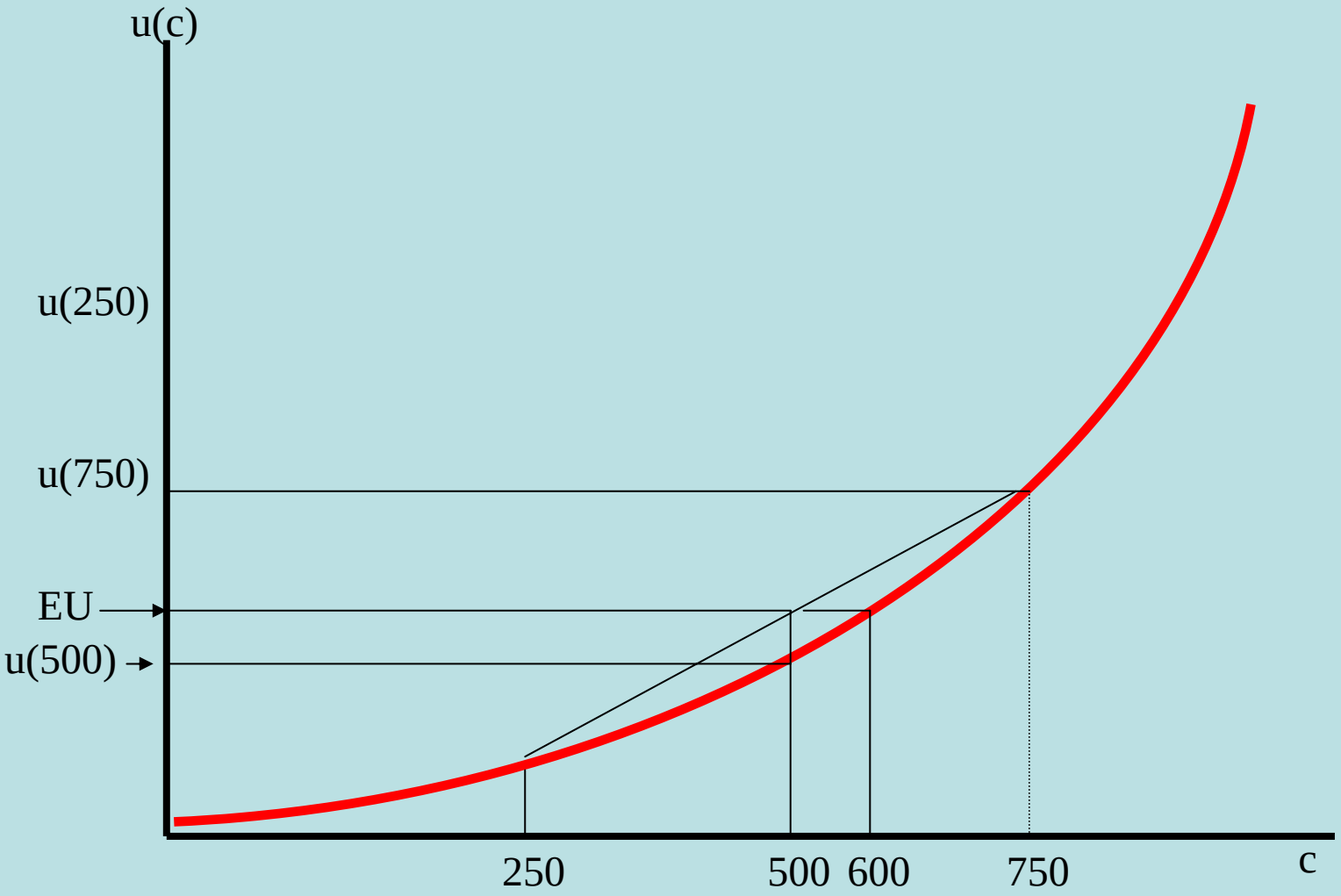
The certainty equivalent of the gamble is $400; the risk premium is $100

A risk neutral person / with constant MU / with a linear utility function will be indifferent between accepting/rejecting a fair gamble



The certainty equivalent of the gamble is $500; the risk premium is $0

A risk loving person / with increasing MU / with a convex utility function will accept a fair gamble



The certainty equivalent of the gamble is $600; the risk premium is -$100