Pure Public Goods

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Private and Public Goods

- A private good satisfies two properties:
  - Its consumption is rivalrous: only one person can consume it
  - Its consumption is excludable: those who do not pay for it are excluded from consuming it

- By contrast, a public good is non-rivalrous in consumption
  - Two or more persons can simultaneously consume the public good
Types of Public Goods

- Pure public goods
  - Consumption is non-rivalrous
  - exclusion is not possible

- Impure public goods: Club goods
  - Consumption is non-rivalrous up to a certain number of users, rivalrous (subject to congestion) thereafter
  - exclusion is possible

- Impure public goods: Common property resources
  - Consumption is non-rivalrous up to a certain number of users, rivalrous thereafter
  - exclusion is not possible
Pure and Impure Public Goods

Pure public good: individual benefit does not depend on number of users

Impure public good: individual benefit does depend on number of users because of congestion

Changes from pure to impure after $n^*$ users
The provision of a public good

There are two goods, private and public and two consumers A and B

$X_A$ and $X_B$ are the quantities of the private good consumed by A and B

The public good is either supplied ($G=1$) or not supplied ($G=0$) at at cost of $C$

If it is supplied, $G_A$ and $G_B$ are the contributions of A and B: $G_A + G_B = C$
Efficient Provision

- The provision of the public good will be efficient if:
  - $u(X_j - G_j, 1) \geq u(X_j, 0)$ for at least one person $j = A, B, >$

- Define the reservation price of $A$ and $B$ as $R_A$ and $R_B$: these are the maximum amounts $A$ and $B$ will pay for the public good.
  - $u(X_A - R_A, 1) = u(X_A, 0)$
  - $u(X_B - R_B, 1) = u(X_B, 0)$

- Necessary and sufficient conditions for $G = 1$ is:
  - $R_A > G_A$ and $R_B > G_B$
  - $R_A + R_B \geq C$
Free Riding

- With a pure public good, exclusion is not possible.
- Consequently, some persons will consume the public good without paying for it: such persons are “free riders”.
- The market will never supply a pure public good because of free riding: market failure!
- Consequently, pure public goods have to be publicly provided and their payment has to be extracted compulsorily through taxes.
Free Riding as a Dominant Strategy

- Benefit of TV = $100
- Cost = $150
- Both A and B can independently decide whether to buy the TV
- Whether B decides to buy or not, A should not buy
- Whether A decides to buy or not, B should not buy
- Free riding is the dominant strategy for A and B

<table>
<thead>
<tr>
<th>A buys</th>
<th>B buys</th>
<th>B free rides</th>
</tr>
</thead>
<tbody>
<tr>
<td>A buys</td>
<td>-50,-50</td>
<td>-50,100</td>
</tr>
<tr>
<td>A free rides</td>
<td>100,-50</td>
<td>0,0</td>
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How Much of a Public Good to Supply?

**X** = private good

**G** = public good

**P**

**C**

**X***

**G***

**Z** (optimal production point)

**D**

**J**

**Slope JJ = slope TT**

**I**

**A**’s utility is held constant on II

**Production Possibility Curve**

**TT is B’s consumption possibility curve**

**JJ is the highest indifference curve B can reach, subject to TT**

**Slope TT = slope PP – slope II**
Samuelson condition

\[ MRS_{XG}^A + MRS_{XG}^B = MRT_{XG} \]

- To produce an extra unit of the public good, technology requires giving up of the MRT_{XG} private good.
- To get an extra unit of the public good, A and B are willing to give up, respectively, MRS_{XG}^A and MRS_{XG}^B of the private good.
- Since they both consume the extra unit of the public good, they are collectively willing to give up \( MRS_{XG}^A + MRS_{XG}^B \) of the private good.
Optimal Conditions for the Supply of Private Goods

$$\text{MRS}_{XY}^A = \text{MRS}_{XY}^B = \text{MRT}_{XY}$$

- X and Y are two private goods
- To produce an extra unit of the private good, Y, technology requires giving up of MRT$_{XY}$ of the private good, X
- To get an extra unit of X, A and B are willing to give up, respectively, MRS$_{XY}^A$ and MRS$_{XY}^B$ of the private good, X
- Since only one of them can consume the extra unit of Y, the equilibrium condition follows
Simplifying the Samuelson Condition

- We may associate the $\text{MRS}_{XG}$ as the marginal benefit a person obtains from the public good, $\text{MB}_G$
- Similarly, we may associate the $\text{MRT}_{XG}$ as the marginal cost of supplying the public good, $\text{MC}_G$
- So, the Samuelson condition becomes

$$\text{MB}_G^A + \text{MB}_G^B = \text{MC}_G$$
Marginal Benefit Pricing

- Although everyone consumes equal quantities of the public good they may obtain different levels of benefit.
- The marginal benefit pricing rule for a pure public good says that people should pay for a public good according to the marginal benefit they obtain from it.
Marginal Benefit Pricing

Marginal Cost = $MC_G$

$MB_G^A + MB_G^B$

Samuelson condition is satisfied

B's price

A's price

$G^*$
Marginal Cost Pricing: under-revelation of benefits, under-provision of quantity

When B correctly reveals his preferences, $G^{**}$ is supplied; when B falsifies his preferences $G^{*}$ is supplied: $G^{**} - G^{*}$ is the under-provision of the public good.
Majority Voting and Public Goods

- Suppose the payment rule is decided in advance of the level of provision
- With three persons, A, B, and C it is (arbitrarily) decided that each will pay the same price:

\[ p^A = p^B = p^C \]
Disagreement about provision levels

At the common price, $p$: A wants $G^A$; B wants $G^B$; C wants $G^C$
Majority Voting: Provision Levels

- A, B, and C are asked to vote on the question: “How much of the public good do you want supplied?”
- They cast their vote for three different levels of provision: $G^A$, $G^B$, and $G^C$
- So, no majority emerges
Majority Voting: Changes to Levels

- Voters are asked if they favour an increase in the supply of the public good (from the current level to a higher level)?
- If a majority votes in favour, the change is effected
- If a majority votes against, the level is left unchanged and there is equilibrium
Majority Voting: Changes to Levels

- A, B, and C are asked if they would like to increase provision beyond GA?
- A votes against; B and C vote in favour and by majority voting provision is increased
- A, B, and C are asked if they would like to increase provision beyond GB?
- A and B vote against; C votes in favour and by majority voting provision is unchanged
- So GB is the equilibrium level of provision
The Median Voter

- Voter B is the median voter
- It is his preference which determines the equilibrium level of provision, $G^B$
- So, under majority rule, the median voter is like a dictator whose personal preference determines the collective outcome
The Median Voter Decides What the Level of Provision Will Be

Note: Preferences are “single-peaked”
The identity of the median voter can change when the benefit curves intersect:

A is the median voter when price = $p_1$; B is the median voter when price = $p_2$
Efficiency and the Median Voter

Inefficient Provision

Efficient Provision

\[ \text{G}^{\text{median}} = \text{G}^* \]

\[ \sum MB = \frac{1}{3} \text{price} \]

\[ \sum MB^B \]

\[ \text{MC} \]

\[ \text{price} \]

\[ 1/3 \text{ price} \]
When will the Median Voter Deliver an Efficient Outcome?

- Suppose there are \( N \) voters, with marginal benefits, \( MB_j, j=1..N \)
- Then average benefit \( \mu = \frac{\sum MB_j}{N} \)
- The preferred supply of the voter with \( MB=\mu \) is given by: \( \mu = \frac{P}{N} = \frac{MC}{N} \)
- So, \( N \times \mu = MC \)
- If the median voter is also the average voter: \( \mu = MB_{\text{median}} \) and we have an efficient outcome
Median versus Average Voter

Under provision of the public good: median voter’s valuation < mean valuation
Problems with Majority Voting

- Majority voting may fail to deliver a stable outcome.
- We ask if, in pair wise comparisons, one alternative is always better than the others.
- If so, we call this alternative the “Condorcet winner.”
- But, there may not be a Condorcet winner.
Three voters: A, B, and C
Three project sizes: S, M, and L
Voters rank projects by size

In a pair-wise comparison
- M beats S (2-1)
- M beats L (2-1)
- M is the Condorcet winner

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<th>A</th>
<th>B</th>
<th>C</th>
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<tbody>
<tr>
<td>S</td>
<td>M</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>S</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>S</td>
<td></td>
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</table>
Vote Cycling

In a pair-wise comparison
- S beats M (2-1)
- M beats L (2-1)
- L beats S (2-1)
- There is no Condorcet winner

Three voters: A, B, and C
Three project sizes: S, M, and L
Voters rank projects by size
Single Peaks and Twin Peaks

Single Peak Preferences

Ranking by Size

Twin Peak Preferences

A’s ranking

B’s ranking

C’s ranking

A’s ranking

B’s ranking

C’s ranking
Appendix on Game Theory

What is a Game?

In a normal-form game, each player chooses a strategy (without knowledge of the strategy chosen by the other players) from his ‘strategy set’ and the pay-off to each player depends on the strategies chosen by all the players.
Some Formalisation

- There are $N$ players, $i=1\ldots N$.
- The set of feasible strategies available to player $i$ is represented by his strategy set $S_i$ ($i=1\ldots N$):
- $s_i \in S_i$ is the strategy chosen by the $i$th player from his set of feasible strategies
- $U^i(s_1, s_2 \ldots s_i \ldots s_N)$ is the pay-off to the $i$th player given his own choice of strategy and the choices made by the other players
- The Game is represented by: $(S_1 \ldots S_N; u^1 \ldots u^N)$
- The solution to the game is: $(s_1^* \ldots s_N^*)$
Dominant Strategy: Prisoners’ Dilemma

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<tr>
<th></th>
<th>Prisoner 2</th>
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<tbody>
<tr>
<td><strong>Prisoner 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Confess</td>
<td>3,3</td>
<td>0,6</td>
</tr>
<tr>
<td>Deny</td>
<td>6,0</td>
<td>1,1</td>
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</tbody>
</table>

P1: if P2 denies, he should confess; if P2 confesses, he should confess
P2: if P1 denies, he should confess; if P1 confesses, he should confess

So, (confess, confess) is the **dominant** strategy

But, it is not the **best** strategy (deny, deny)

1,16,0
Nash Equilibrium

- In a dominant strategy solution, A’s chosen strategy must be represent his best strategy, *regardless of B’s choice*
- However, this is too strong a requirement and dominant strategies may often not exist
- A Nash equilibrium represents a weaker solution
- In a N-E, A’s chosen strategy is the best for the best choice by B and B’s chosen strategy is the best for the best choice by A
Nash Equilibrium Defined

- A’s chosen strategy depends upon B’s choice and B’s chosen strategy depends upon A’s choice
- So \( s_A = R_A(s_B) \) and \( s_B = R_B(s_A) \) where: \( R_A \) and \( R_B \) are A’s and B’s reaction functions
- A Nash equilibrium arises when:
  - Given A’s best choice \( s_A^* \), B’s best choice is \( s_B^* \)
  - Given B’s best choice \( s_B^* \), A’s best choice is \( s_A^* \)
- So, neither A nor B have an incentive to deviate from \( s_A^* \) and \( s_B^* \)
Nash Equilibrium: A Picture

Two Firms

- The dashed arrows represent the path towards the Nash equilibrium.
- At the N-E, no firm has an incentive to change.