Pure Public Goods

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Private and Public Goods

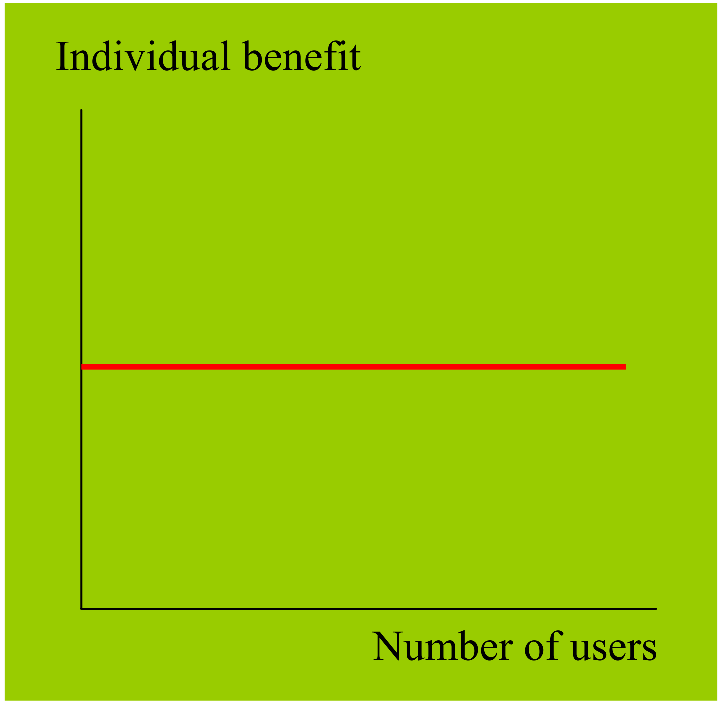
- A private good satisfies two properties:
  - Its consumption is rivalrous: only one person can consume it
  - Its consumption is excludable: those who do not pay for it are excluded from consuming it
- By contrast, a public good is non-rivalrous in consumption
  - two or more persons can simultaneously consume the public good

Types of Public Goods

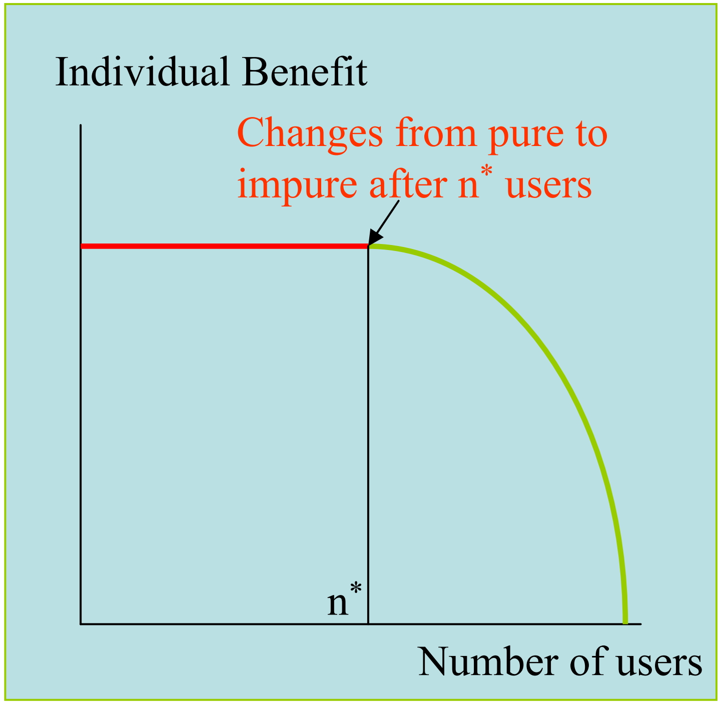
- ❑ Pure public goods
  - ➢ Consumption is non-rivalrous
  - ➢ exclusion is not possible
- ❑ Impure public goods: Club goods
  - ➢ Consumption is non-rivalrous up to a certain number of users, rivalrous (subject to congestion) thereafter
  - ➢ exclusion is possible
- ❑ Impure public goods: Common property resources
  - ➢ Consumption is non-rivalrous up to a certain number of users, rivalrous thereafter
  - ➢ exclusion is not possible

Pure and Impure Public Goods

Pure public good: individual benefit does not depend on number of users



Impure public good: individual benefit does depend on number of users because of congestion



The provision of a public good

- There are two goods, private and public and two consumers A and B
- X_A and X_B are the quantities of the private good consumed by A and B
- The public good is either supplied (G=1) or not supplied (G=0) at at cost of $C
- If it is supplied, G_A and G_B are the contributions of A and B: $G_A+G_B=C$

Efficient Provision

- The provision of the public good will be efficient if:
  - $u(X_j - G_j, 1) \geq u(X_j, 0)$ $j=A,B$, > for at least one person
- Define the *reservation price* of A and B as R_A and R_B: these are the *maximum* amounts A and B will pay for the public good
  - $u(X_A - R_A, 1) = u(X_A, 0)$
  - $u(X_B - R_B, 1) = u(X_B, 0)$
- Necessary and sufficient conditions for G=1 is:
  - $R_A > G_A$ and $R_B > G_B$
  - $R_A + R_B \geq C$

Free Riding

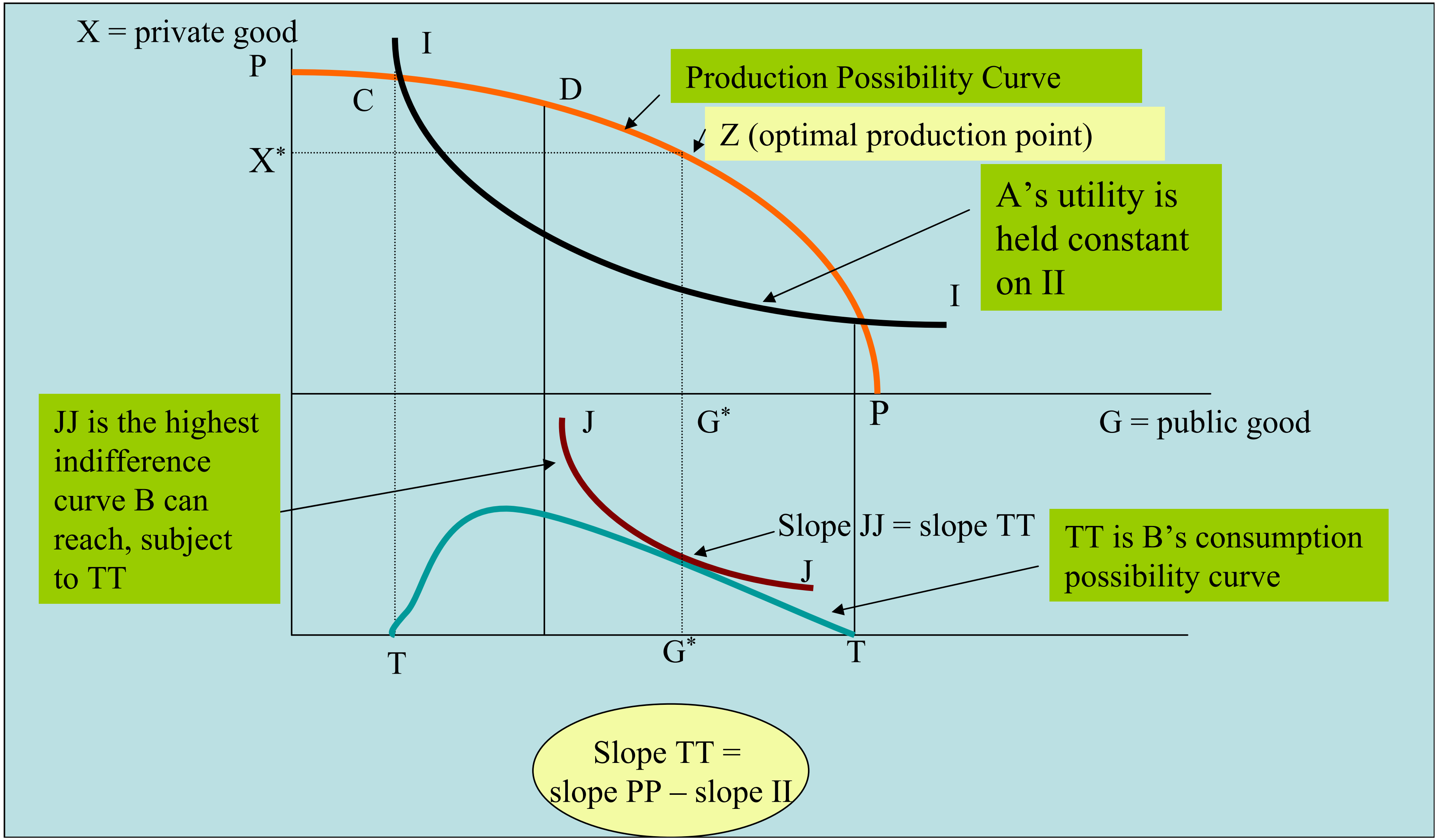
- With a pure public good, exclusion is not possible
- Consequently, some persons will consume the public good without paying for it: such persons are "free riders"
- The market will never supply a pure public good because of free riding: market failure!
- Consequently, pure public goods have to be publicly provided and their payment has to be extracted compulsorily through taxes

Free Riding as a Dominant Strategy

	B buys	B free rides
A buys	-50,-50	-50,100
A free rides	100,-50	0,0

- Benefit of TV = $100
- Cost = $150
- Both A and B can independently decide whether to buy the TV
- Whether B decides to buy or not, A should not buy
- Whether A decides to buy or not, B should not buy
- Free riding is the ***dominant*** strategy for A and B

How Much of a Public Good to Supply?



Samuelson condition

$MRS_{XG}^{A} + MRS_{XG}^{B} = MRT_{XG}$

- To produce an extra unit of the public good, technology requires giving up of the MRT_{XG} private good
- To get an extra unit of the public good, A and B are willing to give up, respectively, MRS_{XG}^{A} and MRS_{XG}^{B} of the private good
- Since they both consume the extra unit of the public good, they are collectively willing to give up $MRS_{XG}^{A} + MRS_{XG}^{B}$ of the private good

Optimal Conditions for the Supply of Private Goods

$MRS_{XY}^{A} = MRS_{XY}^{B} = MRT_{XY}$

- X and Y are two private goods
- To produce an extra unit of the private good, Y, technology requires giving up of MRT_{XY} of the private good, X
- To get an extra unit of X, A and B are willing to give up, respectively, MRS_{XY}^{A} and MRS_{XY}^{B} of the private good, X
- Since only one of them can consume the extra unit of Y, the equilibrium condition follows

Simplifying the Samuelson Condition

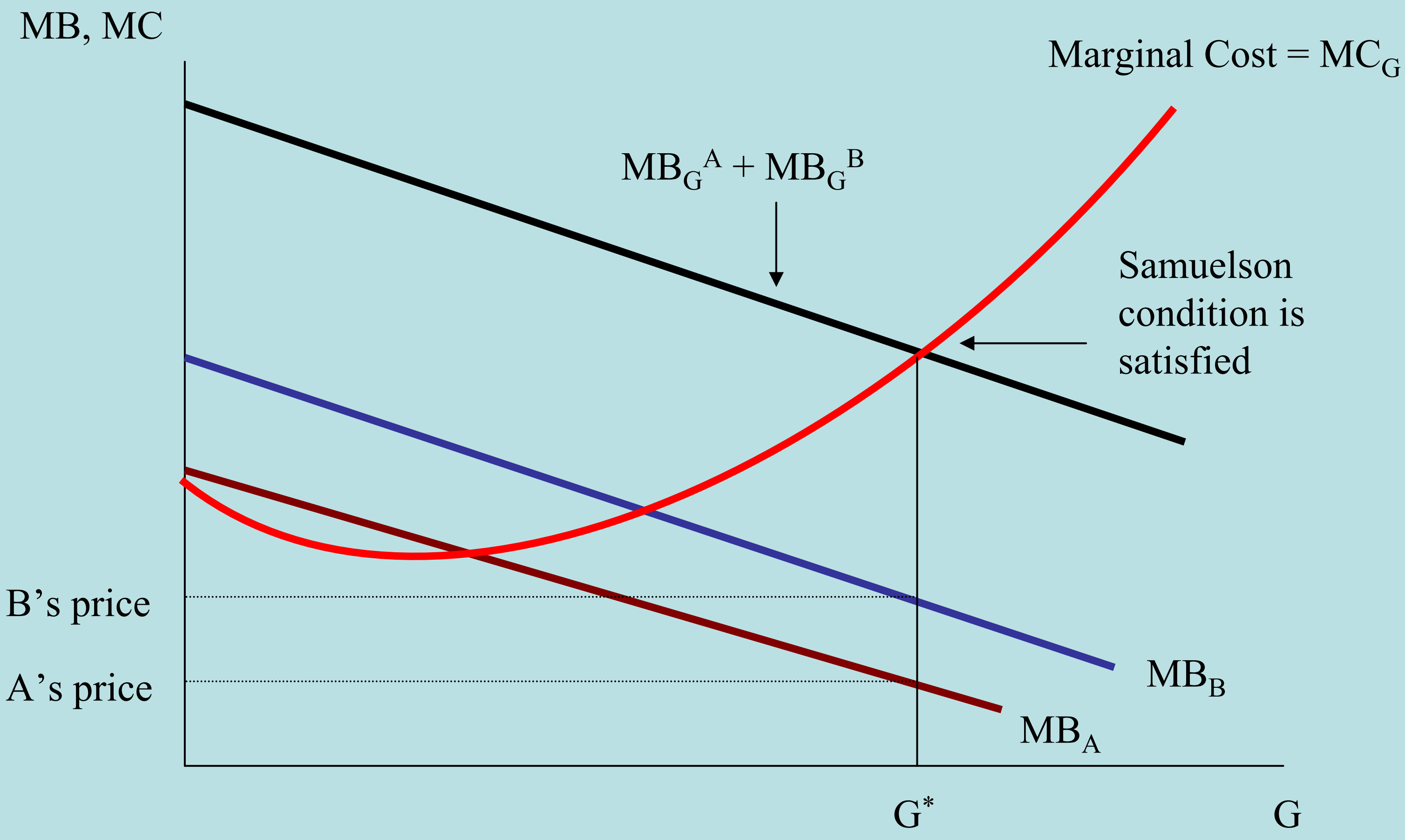
- We may associate the MRS_{XG} as the marginal benefit a person obtains from the public good, MB_G
- Similarly, we may associate the MRT_{XG} as the marginal cost of supplying the public good, MC_G
- So, the Samuelson condition becomes

$$MB_G^A + MB_G^B = MC_G$$

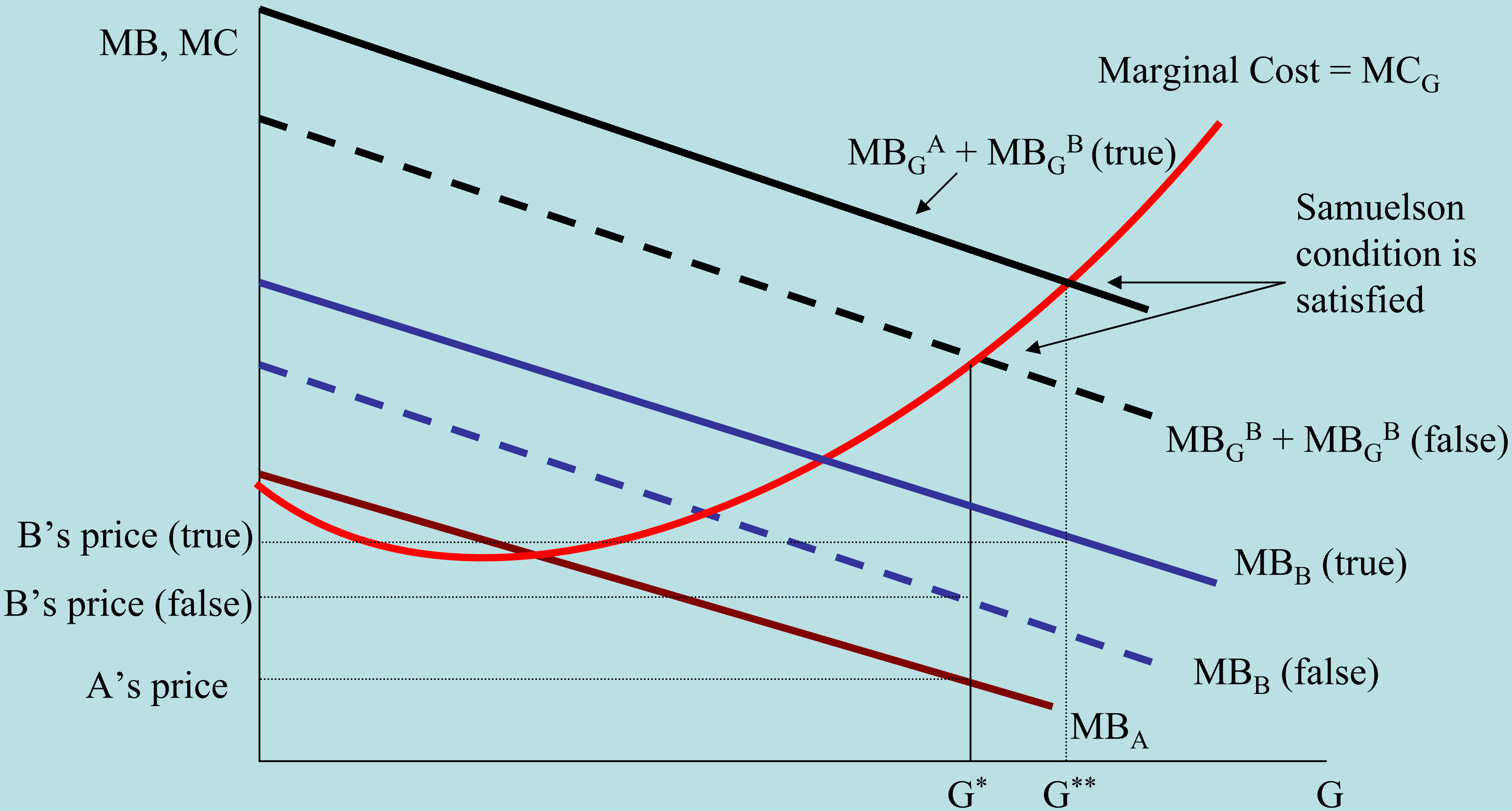
Marginal Benefit Pricing

- Although everyone consumes equal quantities of the public good they may obtain different levels of benefit
- The marginal benefit pricing rule for a pure public good says that people should pay for a public good according to the marginal benefit they obtain from it

Marginal Benefit Pricing



Marginal Benefit Pricing: under-revelation of benefits, under-provision of quantity



When B correctly reveals his preferences, G^{**} is supplied; when B falsifies his preferences G^* is supplied: $G^{**} - G^*$ is the under-provision of the public good

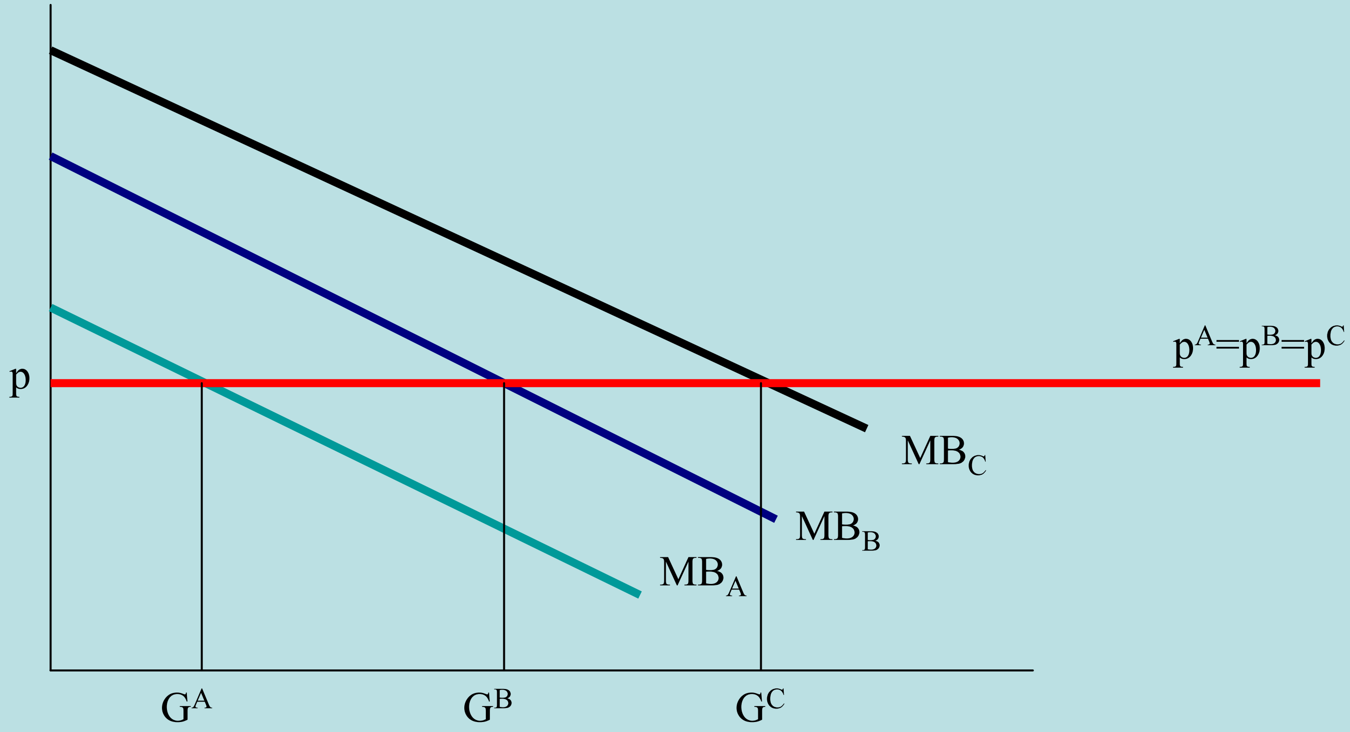
Majority Voting and Public Goods

- Suppose the payment rule is decided in advance of the level of provision
- With three persons, A, B, and C it is (arbitrarily) decided that each will pay the same price:

$$p^A = p^B = P^C$$

Disagreement about provision levels

At the common price, p: A wants G^A; B wants G^B; C wants G^C



MajorityVoting: Provision Levels

- A, B, and C are asked to vote on the question: “How much of the public good do you want supplied?”
- They cast their vote for three different levels of provision: G^A, G^B, and G^C
- So, no majority emerges

Majority Voting: Changes to Levels

- Voters are asked if they favour an increase in the supply of the public good (from the current level to a higher level)?
- If a majority votes in favour, the change is effected
- If a majority votes against, the level is left unchanged and there is equilibrium

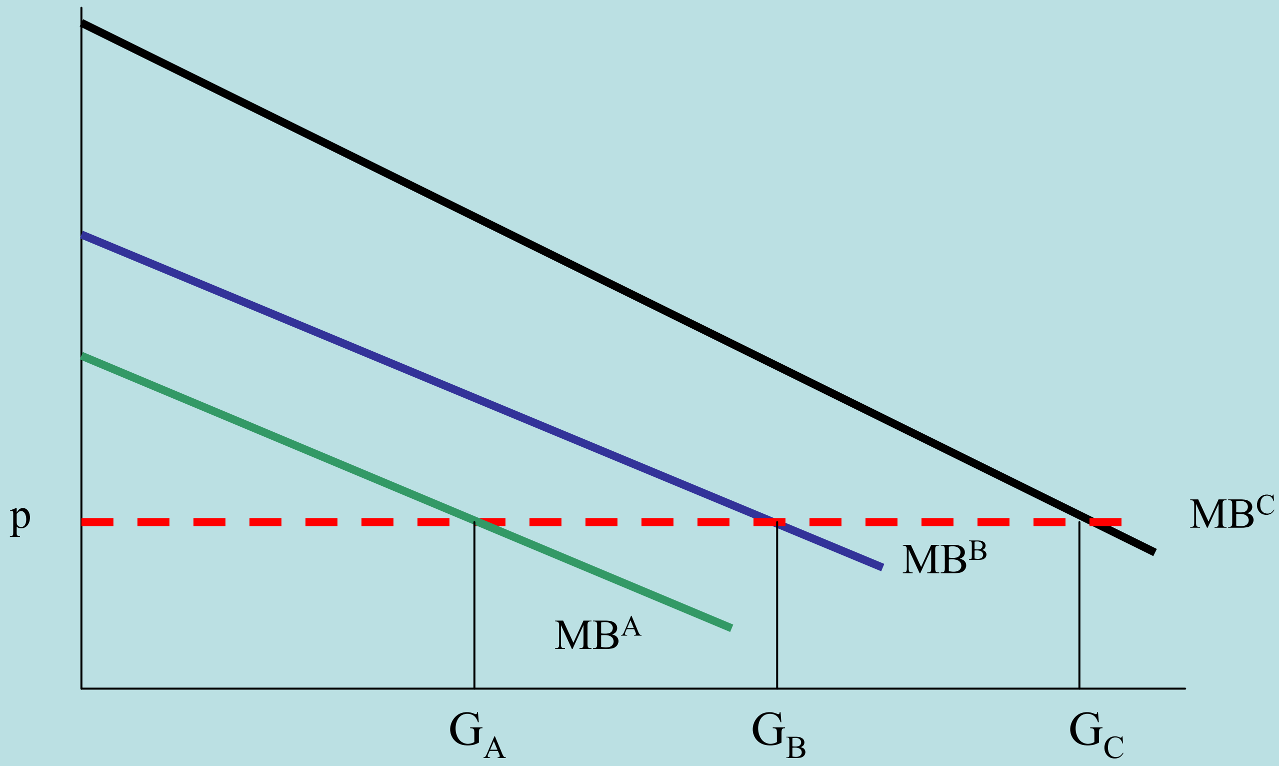
Majority Voting: Changes to Levels

- A, B, and C are asked if they would like to increase provision beyond G^A?
- A votes against; B and C vote in favour and by majority voting provision is increased
- A, B, and C are asked if they would like to increase provision beyond G^B?
- A and B vote against; C votes in favour and by majority voting provision is unchanged
- So G^B is the equilibrium level of provision

The Median Voter

- Voter B is the median voter
- It is his preference which determines the equilibrium level of provision, G^B
- So, under majority rule, the median voter is like a dictator whose personal preference determines the collective outcome

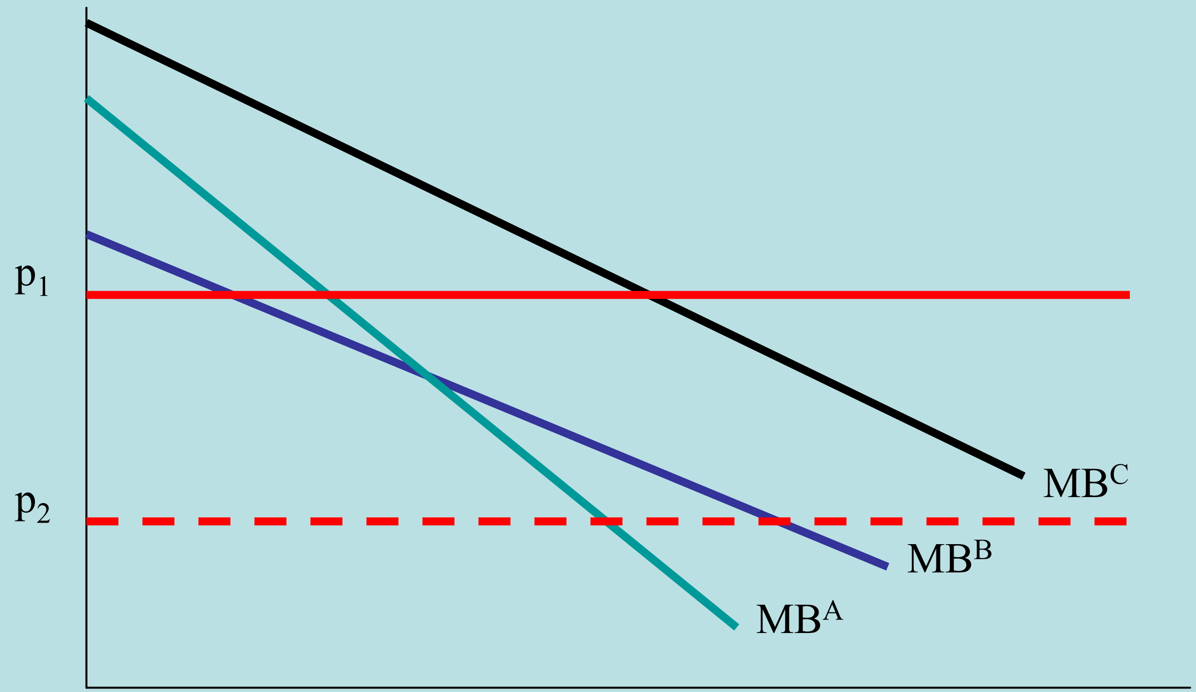
The Median Voter Decides What the Level of Provision Will Be



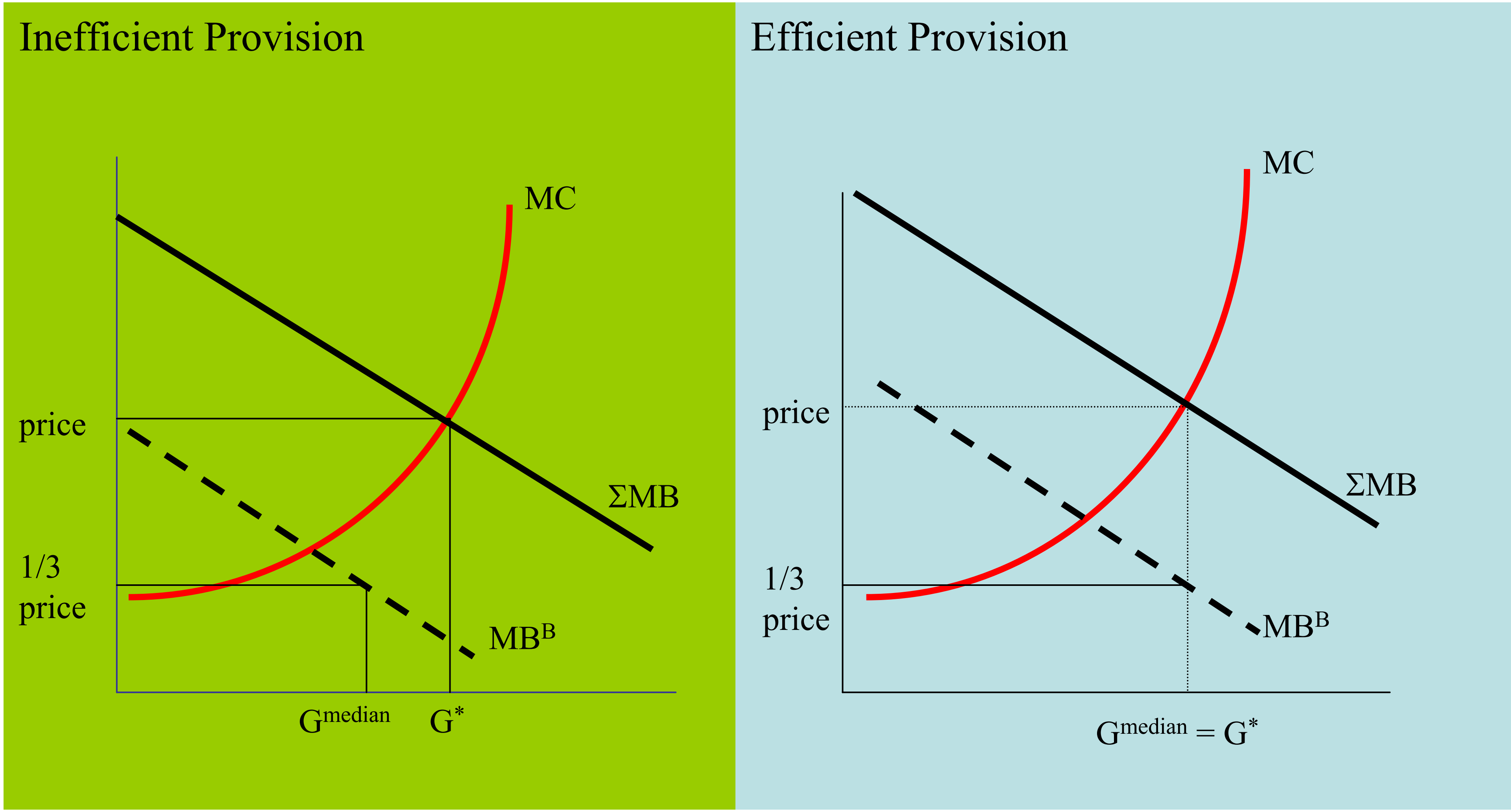
Note: Preferences are “single-peaked

The identity of the median voter can change when the benefit curves intersect:

A is the median voter when price = p_1; B is the median voter when price = p_2



Efficiency and the Median Voter

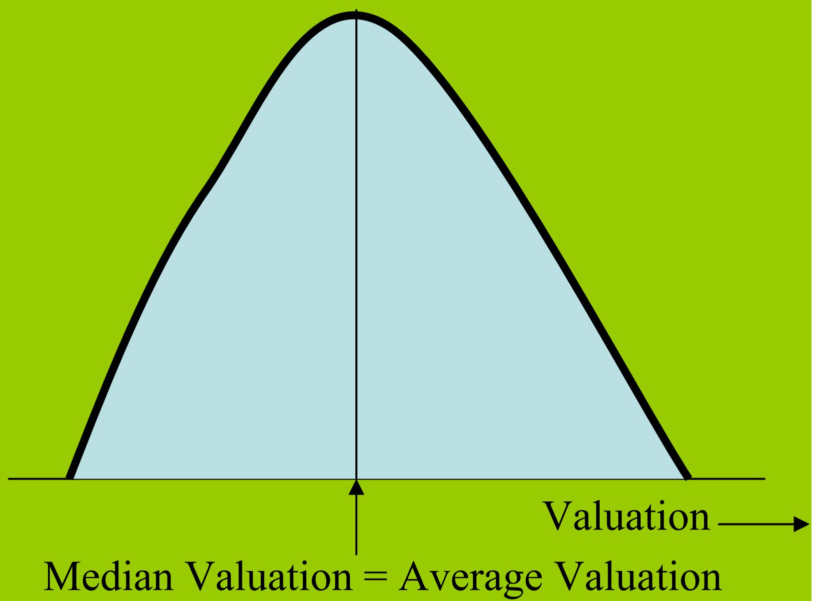


When will the Median Voter Deliver an Efficient Outcome?

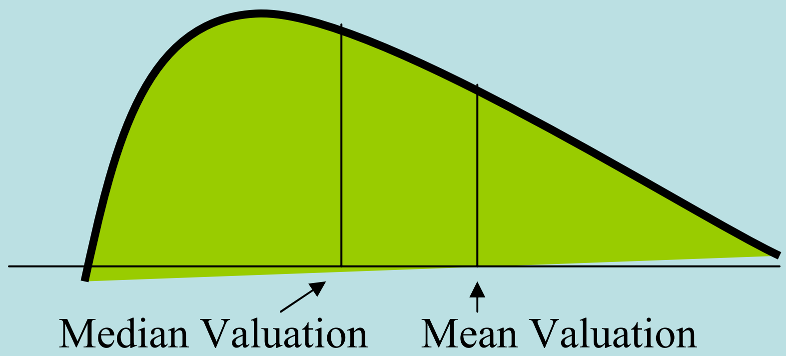
- Suppose there are N voters, with marginal benefits, MB_j, j=1..N
- Then average benefit $\mu=\Sigma MB_j/N$
- The preferred supply of the voter with $MB=\mu$ is given by: $\mu=P/N=MC/N$
- So, $N\times\mu = MC$
- If the median voter is also the average voter: $\mu=MB^{median}$ and we have an efficient outcome

Median versus Average Voter

Efficient Outcome: Median Voter is Average Voter



Under provision of the public good: median voter's valuation < mean valuation



Problems with Majority Voting

- Majority voting may fail to deliver a stable outcome
- We ask if, in pair wise comparisons, one alternative is always better than the others
- If so, we call this alternative the "Condorcet winner"
- But, there may not be a Condorcet winner

Condorcet Winners

Three voters: A, B, and C
Three project sizes: S, M, and L
Voters rank projects by size

A	B	C
S	M	L
M	S	M
L	L	S

In a pair-wise comparison

- M beats S (2-1)
- M beats L (2-1)
- M is the Condorcet winner

Vote Cycling

Three voters: A, B, and C
Three project sizes: S, M, and L
Voters rank projects by size

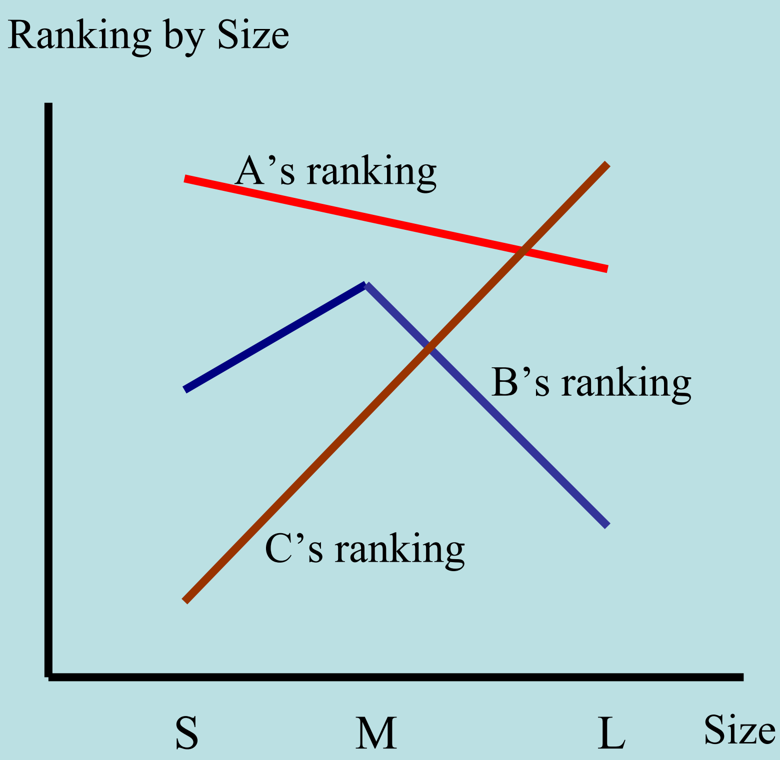
A	B	C
S	M	L
M	L	S
L	S	M

In a pair-wise comparison

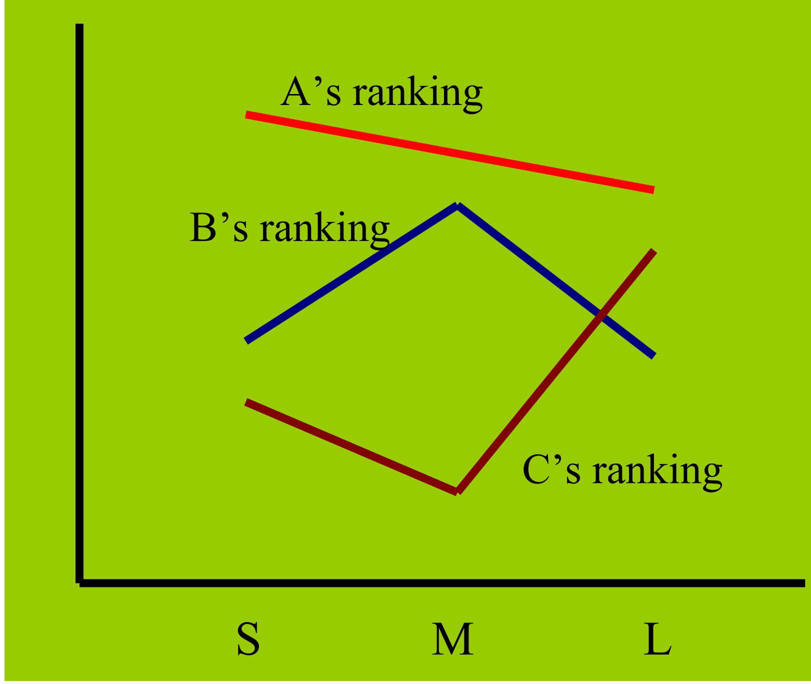
- S beats M (2-1)
- M beats L (2-1)
- L beats S (2-1)
- There is no Condorcet winner

Single Peaks and Twin Peaks

Single Peak Preferences



Twin Peak Preferences



Appendix on Game Theory

What is a Game?

In a normal-form game, each player chooses a strategy (without knowledge of the strategy chosen by the other players) from his ‘strategy set’ and the pay-off to each player depends on the strategies chosen by *all* the players.

Some Formalisation

- There are N players, i=1…N.
- The set of feasible strategies available to player i is represented by his *strategy set* S_i (i=1…N):
- $s_i \in S_i$ is the strategy chosen by the ith player from his set of feasible strategies
- $U^i(s_1,s_2..s_i..s_N)$ is the pay-off to the ith player given his own choice of strategy *and the choices made by the other players*
- The Game is represented by: $(S_1..S_N; u^1..u^N)$
- The solution to the game is: $(s_1^*…s_N^*)$

Dominant Strategy:Prisoners' Dilemma

Prisoner 2

Prisoner 1

	Confess	**Deny**
Confess	3,3	0,6
Deny	6,0	1,1

P1: if P2 denies, he should confess; if P2 confesses, he should confess

P2: if P1 denies, he should confess; if P1 confesses, he should confess

So, (confess, confess) is the ***dominant*** strategy

But, it is not the ***best*** strategy (deny, deny)

Nash Equilibrium

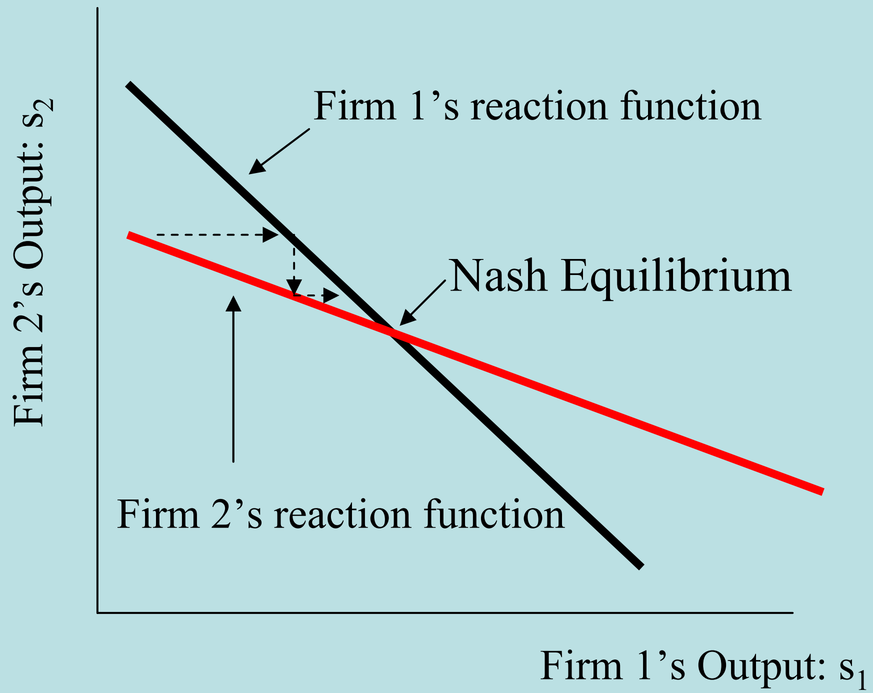
- In a dominant strategy solution, A's chosen strategy must be represent his best strategy, ***regardless of B's choice***
- However, this is too strong a requirement and dominant strategies may often not exist
- A Nash equilibrium represents a weaker solution
- In a N-E, A's chosen strategy is the best for the best choice by B and B's chosen strategy is the best for the best choice by A

Nash Equilibrium Defined

- A's chosen strategy depends upon B's choice and B's chosen strategy depends upon A's choice
- So $s_A = R_A(s_B)$ and $s_B = R_B(s_A)$ where: R_A and R_B are is A's and B's reaction functions
- A Nash equilibrium arises when:
   - Given A's best choice s_A^*, B's best choice is s_B^*
   - Given B's best choice s_B^*, A's best choice is s_A^*
- So, neither A nor B have an incentive to deviate from s_A^* and s_B^*

Nash Equilibrium: A Picture

Two Firms



- The dashed arrows represent the path towards the Nash equilibrium
- At the N-E, no firm has an incentive to change