Market Failure
An Economic Analysis of its Causes and Consequences

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1. Introduction

Much of economic theory of the textbook variety is a celebration of the free market system. This celebration has two parts. First, the operation of the price system, in the context of competitive markets, leads to balance between the demand and supply of the different goods and services traded. In other words, flexible prices result in competitive markets clearing. Second, the market-clearing equilibrium - brought about through flexible prices and competitive markets - is a "good thing" in the sense that it is also a point of economic efficiency[1]. In other words competitive outcomes are also efficient ones. The fact that competition leads to efficiency is known as the *First Fundamental Theorem of Welfare Economics*[2].

These results - which are, of course, a vindication of Adam Smith's intuition about the existence of an "invisible hand" bringing consistency and order to the chaos of individual actions - would be remarkable in themselves. But there is more. The efficient outcome will have been brought about through parsimony in the use of information; the only things that individuals, in making their supply/demand decisions, need to know are the prices of the different commodities. Furthermore, since the efficient outcome is the result of firms and consumers acting "selfishly", by looking only to their own interests, it is "incentive compatible" in the sense that its existence does not depend upon altruistic behaviour. Lastly, not only will competitive markets lead to an efficient outcome, but any efficient outcome that one might desire can be attained through the operation of competitive markets. This last statement – known as the *Second Fundamental Theorem of Welfare Economics* - is a very powerful result for it says that if one does not like the particular efficient outcome (perhaps because there were great inequalities associated with it) that resulted from the operation of competitive markets in a specific context, then all is not

[1] Economists regard an outcome as being "efficient" if there no other another outcomes in which, relative to the original outcome, some persons are better off without anyone being worse off. If such "better" outcomes exist, then the original outcome is termed inefficient.
[2] See Arrow and Hahn (1971), for an authoritative account of how competitive economies work.

lost. In such a situation all that is required is to specify a different, more desirable, outcome and to modify the context suitably; competitive markets operating in the new context would then lead to that outcome.

Against the background of these results the government does not have much of a role. If economic outcomes were not socially desirable then one role for government would be to change the context within which markets operated. This context is provided by the initial endowments with which individuals are equipped for trade in the market. For example, persons who were wealthy or who possessed skills and education would be better equipped for trade than the poor and the unskilled and hence would benefit disproportionately from market outcomes. If endowments were unfairly distributed then market outcomes, notwithstanding the fact that they were efficient, would also be unfair.

Thus, within the framework of market sovereignty, *redistribution* - whereby initial endowments were altered in order to prevent grossly inequitable outcomes - would be an acceptable role for government. Even here, its role would be limited by the injunction that, in the pursuit of redistributive objectives, the government should not, by distorting incentives, pervert the free functioning of markets. Since this injunction could only be satisfied through the highly infeasible instrument of "lump-sum" taxes and transfers (that is, all "rich" persons pay, for example, $100 each, and all "poor" persons receive the same, irrespective of their wealth or poverty), in practice the redistributive role for government would be non-existent.

Although proponents of free markets concede that government might legitimately have a say, however circumscribed, in the sphere of distribution (that is, in terms of who receives how much) it would deny government any role, other than a purely facilitating one, in the spheres of production and of allocation (that is, in terms of deciding what, how much of, and by whom, commodities are to be produced.

In terms of ‘by whom to produce’, the basic choice is between production by the private sector and production by the public sector. Economists who believe in the free functioning of markets would argue that the most useful role that government could play in this regard would be to abdicate its productive responsibilities in favour of the private sector - a process known as *privatisation*. Such economists would, in similar vein, argue that the most useful contribution that governments could make to allocative decisions (relating to what, and how much, to produce) would be to remove *market imperfections*. These imperfections, which prevent markets from functioning properly, are associated with an absence of competition (for example, through the existence of monopolies) or with the presence of barriers to price flexibility (for example, through price-support mechanisms like minimum wage legislation). The task of government would then be to take the necessary steps to ensure that all impediments to the proper functioning of markets were removed.

Economists who are not content with this purely passive role for public policy, point to numerous real-world instances where, notwithstanding the existence of competition and price flexibility, markets fail to deliver on efficiency. (Indeed as Solow (1993) has pointed out, many of the young stars of economics, of the past twenty years, made their mark by going beyond the simple competitive model and considering the consequences of dropping some of its restrictive assumptions). In the presence of such cases of *market failure*, they would argue, governments have no alternative but to intervene actively to help markets overcome these difficulties. Indeed Stiglitz (1989) has argued that , contrary to the traditional view that market failures are the exception, such failures may be so pervasive as to be the norm.

However, it is not at all obvious that government will necessarily succeed where markets have failed. Consequently, not all cases of market failure will be amenable to correction through government action. The key to effective government intervention, therefore, lies not in demonstrating the existence of market failures (and thereby establishing a rationale for government intervention) but rather one of identifying situations where such failures are of the kind that would make intervention worthwhile.

2. The Conditions for Efficiency

Efficiency in production arises when factors of production are allocated between the production of different commodities in such a way that a different allocation would not produce more of some commodities without also producing less of some other commodities. The necessary and sufficient condition for efficiency in production, when there are two factors of production, labour and capital - whose quantities are represented by *L* and *K*, respectively – used in producing two commodities, whose quantities are represented by *X* and *Y*, respectively, is:

$$MRTS_{LK}^{X} = MRTS_{LK}^{Y} \qquad (1)$$

where: MRTS is *the marginal rate of technical substitution* between labour and capital[3].

Efficiency in consumption arises when the available quantities of the different commodities are distributed between the different consumers in such a way that a different distribution would not lead some consumers to be better off without making some other consumers worse off. The necessary and sufficient condition for efficiency in consumption, when fixed quantities of two commodities, denoted *X* and *Y*, are to be distributed between two consumers, *A* and *B* is:

$$MRS_{XY}^{A} = MRS_{XY}^{B} \qquad (2)$$

where: MRS is *the marginal rate of substitution* between the two commodities. labour and capital[4].

Efficiency in production is illustrated in Figure 1: each point on the *efficiency locus* in Figure 1 represents a point of tangency between isoquants and can be mapped onto a point on the production possibility curve in Figure 2: W→D and V→C. Figure 3 illustrates efficiency in consumption and Figure 4 shows the mapping of the efficient points in consumption (of Figure 3) onto a utility possibility curve.

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[3] MRTS is the reduction in the amount of capital needed when an additional unit of labour is used in the production of a given amount of output.

[4] MRS is the amount of one commodity a consumer is prepared to give up, to get an additional unit of another commodity, utility remaining unchanged.

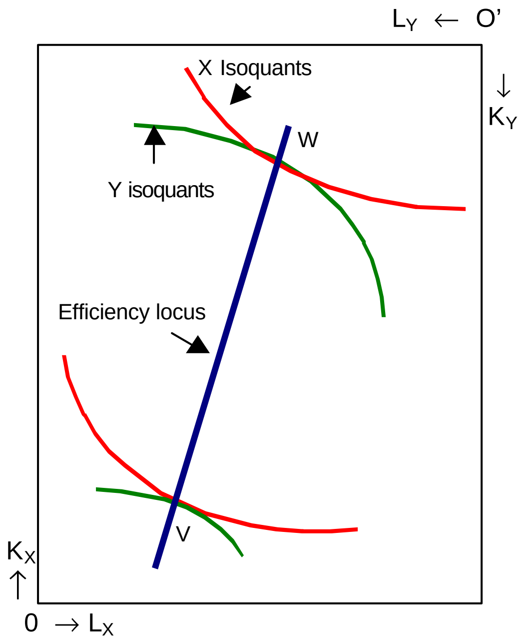

Figure 1:
The Edgeworth Box in Production

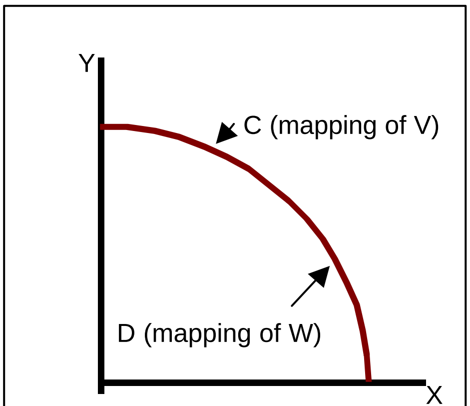

Figure 2:
The Production-Possibility Curve

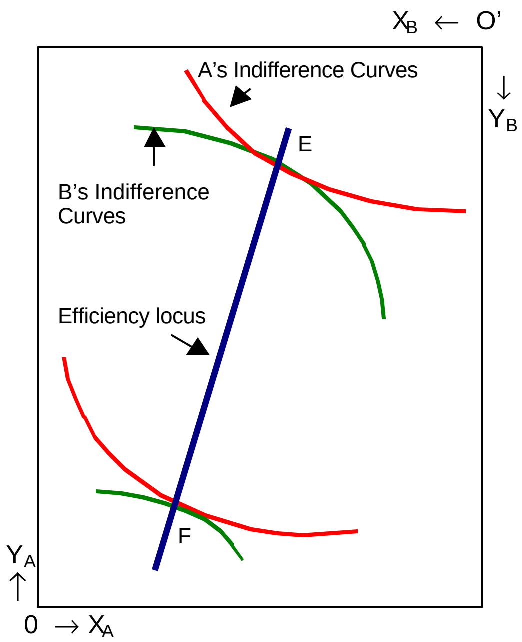

Figure 3:
The Edgeworth Box in Consumption

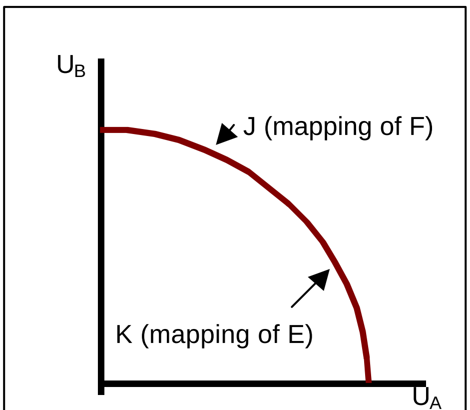

Figure 4:
The Utility-Possibility Curve

Each point on the production possibility curve represents an efficient allocation of labour and capital in production; given a point on the production possibility curve – representing the total quantities produced of each of the two commodities - each point on the utility possibility curve represents an efficient distribution of the two commodities in consumption.

Allocative efficiency occurs when the “right” amounts of the two commodities are produced or, in other words, the economy is at the “right” point on its production possibility curve. The necessary and sufficient condition for allocative efficiency is:

$$MRS_{XY}^{A} = MRS_{XY}^{B} = MRT_{XY} \qquad (3)$$

where: MRT_{XY} is the *marginal rate of transformation* between the two commodities[5]. The point G in Figure 5, below, is a point of allocative efficiency: at G, the slope of the production possibility curve (MRT_{XY}) is equal to the (common) slopes of the indifference curves ($MRS_{XY}^{A} = MRS_{XY}^{B}$).

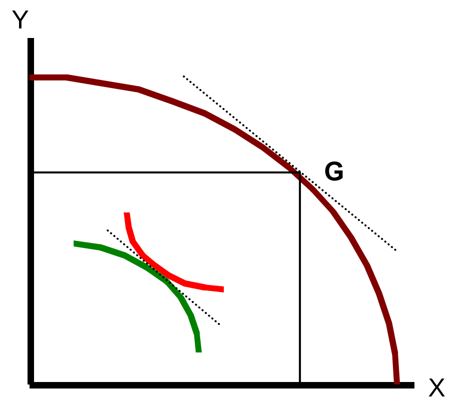

Figure 5:
Allocative Efficiency

See the Mathematical Appendix for a derivation of the efficiency conditions of equation (3).

[5] Suppose $MRT_{XY}=\alpha$ and $MRS_{XY}=\beta$, $\alpha>\beta$. Then if A loses one unit of X, because one unit less of X is produced, A will need to be compensated by β units of Y; but producing one unit less of X, will increase output of Y by $\alpha>\beta$. So, after A has been compensated, following a unit reduction in the production of X, there will be a ‘surplus’ of Y.

3. The Fundamental Theorems of Welfare Economics

The First Theorem: Every Competitive Equilibrium is an Efficient Outcome

Under a competitive equilibrium, every consumer faces the same prices. Therefore, equilibrium for consumers A and B occurs when:

$MRS^A_{XY} = p_X / p_Y$ and $MRS^B_{XY} = p_X / p_Y \Rightarrow MRS^A_{XY} = MRS^B_{XY}$.

Similarly, each producer faces the same input prices. Therefore, equilibrium for each producer occurs when: $MRTS^X_{LK} = q_K / q_L$ and $MRTS^Y_{LK} = q_K / q_L$

$\Rightarrow MRTS^X_{LK} = MRTS^Y_{LK}$ for $\forall i, j = 1...N$

Lastly, $MRT_{XY} = MC_X / MC_Y$ and, under competitive conditions:

$p_X = MC_X$ and $p_Y = MC_Y \Rightarrow p_X / p_Y = MRT_{XY} = MRS^A_{XY} = MRS^B_{XY}$ so that the conditions for allocative efficiency are satisfied.

Second Theorem: If preferences and technology are convex, then every Pareto optimal outcome can be represented as a competitive equilibrium for some pattern of initial endowments and some vector of prices.

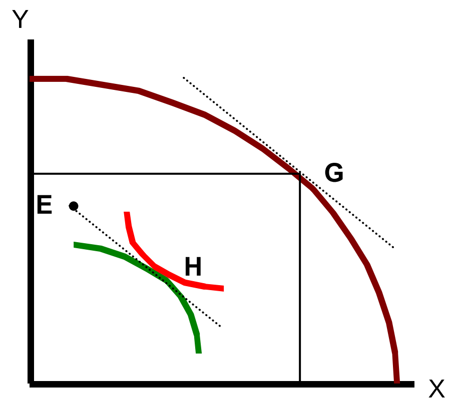

Figure 6:
The Second Theorem

In Figure 6, consumers with an initial endowment at E, when faced with prices represented by the slope of the dashed line, will arrive at the efficient point, H.

Similarly, producers will produce at G. Consequently, the Pareto optimal outcome at G and H can be attained through the competitive process.

4. Market Failure due to Monopoly

Suppose that commodity X is produced under monopolistic conditions, while Y is produced under competitive conditions. Then, $p_X > MC_X, p_Y = MC_Y$. This implies that: $MRT_{XY} = \frac{MC_X}{MC_Y} < \frac{p_X}{p_Y}$. Since, in consumption: $MRS^A_{XY} = MRS^B_{XY} = \frac{p_X}{p_Y}$ it follows:

$$MRT_{XY} < MRS^A_{XY} = MRS^B_{XY} \qquad (4)$$

Consequently, the rate at which producers are able to transform commodity Y into commodity X *is less* than the rate at which consumers are willing to substitute commodity X for commodity Y. Consequently, labour and capital are being misallocated since *too little* of X is being produced[6].

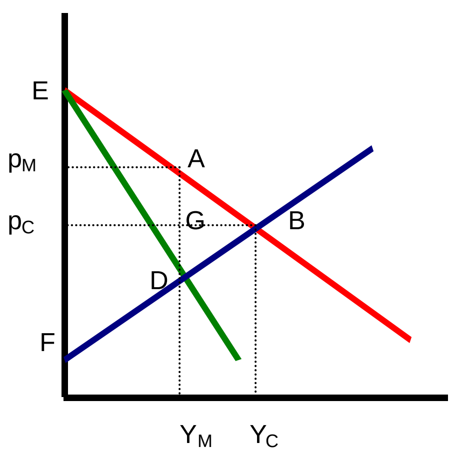

Figure 7:
Welfare Losses from Monopoly

[6] Consumers are prepared to give up MRS_{XY} units of Y for an additional unit of X. Producers, however, have to reduce production of Y by only MRT_{XY} units in order to produce an additional unit of X. So consumers would be better off if an additional unit of X is produced since they are willing to give up more of Y than they need to.

The loss in consumers' surplus is $p_M ABp_C = p_M AGp_C$ + AGD. The gain in producer's surplus is $Fp_M AD$ - $p_c BD = p_M AGp_C$ – DGB. So, the *net loss due to monopoly* is ($p_M AGp_C$ + AGD) –($p_M AGp_C$ – DGB) = AGD + DGB = ABD. The area of the triangle ABD – whose area measures the net loss from monopoly – is known as the *deadweight loss* from monopoly (Harberger, 1954).

Natural Monopoly

A 'natural monopoly' arises when average cost declines over the relevant range of demand. When a natural monopoly exists for a particular commodity, it is the price elasticity for that commodity which determines whether, or not, the existence of the natural monopoly has implications for public policy.

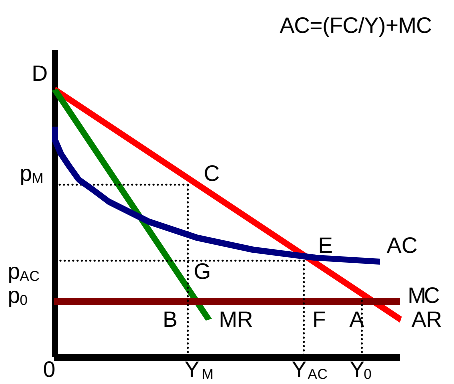

Figure 8:
Natural Monopoly

Source: Weimer and Vining (1999), p. 102

In Figure 8, AC = (FC/Y) +(VC/Y). It is assumed that MC=VC/Y=constant so that: AC=(FC/Y)+MC. FC=$p_{AC}EFp_0$. The profit-maximising output and price are Y_M and p^M, respectively. Monopoly profit is total revenue ($0p_M CY_M$) less VC ($0p_0 BY_M$) less FC ($p_{AC}EFp_0$). Hence, monopoly profit = $p_0 p_M CB$ – FC.

At the competitive output, p_0=MC and the firm makes a loss equal to FC when it is producing the competitive output, Y_0. Comparing the competitive to the monopoly outcome, profit under monopoly is higher by $p_0 p_M CB$, but consumers' surplus under monopoly is lower by $p_M CAp_0 = p_0 p_M CB$ + ABC. Hence, the net loss from the natural monopoly is the area of the triangle **ABC**.

However, public policy regulation which forced the monopoly to price competitively would drive it out of business since at p_0, Y_0 it would not be covering its costs. Suppose the firm were allowed to price at average, instead of marginal cost. Then the regulated price and output are p_{AC} and Y_{AC}, respectively. The gain in consumers' surplus is now $p_{AC}p_M$CE = $p_{AC}p_M$CG + GCE and loss in profits to the producer is simply monopoly profit: p_0p_MCB – FC = $p_{AC}p_M$CG + P_0p_{AC}GB – (P_0p_{AC}GB + BGEF). So, comparing the gain in consumers' surplus to the loss in profits, the net gain from (average cost) regulation, *compared to monopoly*, is GCE + BGEF = **BCEF**. Consequently, in moving from efficient pricing (p_0, Y_0) to average cost pricing (p_{AC}, Y_{AC}) the net loss is only **AEF**.

Industries with low barriers to entry and decreasing average costs are said to be *contestable* markets: there is competition *for* the market even though there is no competition *in* the market (Baumol, Panzar and Willig, 1982). A natural monopolist who is in a contestable market is likely to price near average cost in order to deter potential entrants. An important issue in the context of contestable markets is the role of installed capital as a barrier to entry. For example, the postal service may be a decreasing returns industry but, in the absence of initial capital investment as a deterrent to entry, it may be highly contestable (Sorkin, 1980).

X-Inefficiency

Leibenstein (1976) and Franz (1988) developed the concept of *X-inefficiency* to describe a situation in which a firm, because of lack of competition, does not operate at the minimum costs that are technically feasible. For example, a charge made against publicly owned industries is that they do not have incentives to operate at minimum cost. Consequently, privatising an industry (or 'contracting out' certain parts of its operation) may deliver benefits in terms of lower costs of production.

Figure 9 shows that, in the absence of *X*-inefficiency, the gain in consumers' surplus in moving from the monopoly (p_m, Y_m) to the competitive (p_c, Y_c)

outcome is: $p_c p_m$BA and the loss in monopoly profits is: $p_c p_m$BC. So the net social gain from the move is the area **ABC**. Under X-inefficiency, the MC curve is higher and the net social gain of the move from X-inefficiency monopoly to the competitive outcome is the area **AGE.** Consequently, the net social gain in moving from '*X*-inefficient' monopoly to 'minimum cost' monopoly is **GEBC**.

The *X*-inefficient monopoly in producing Y_X, incurs an 'extra' cost of $p_c MC_x$FG. If this extra cost represents the deployment of real resources (more workers than needed) then it should be regarded as a social loss and added to GEBC, the deadweight loss from *X*-inefficiency. On the other hand, if the extra cost of $p_c MC_x$FG arises because of higher salaries and wages then it should be regarded as rent.

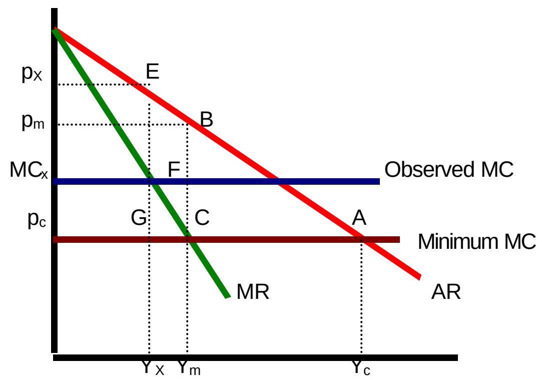

Figure 9:
X-Inefficiency in Monopoly

Source: Weimer and Vining (1999), p. 106

5. Market Failure Due to Information Asymmetry

Information asymmetry refers to the fact that the buyer and the seller of a commodity may have different amounts of information about that commodity's attributes.

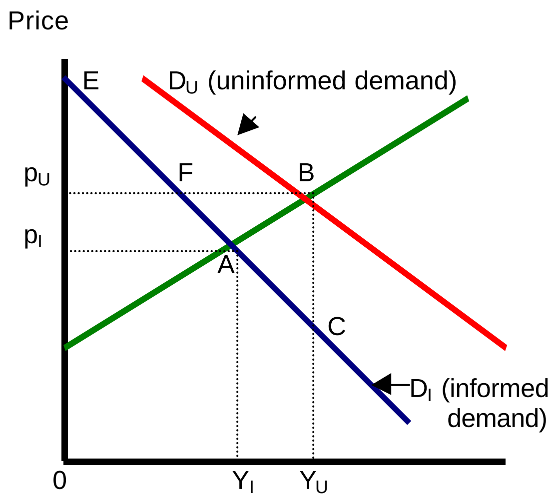

Figure 10:
Market Failure From Information Asymmetry

Source: Weimer and Vining (1999), p. 108

In Figure 10, D_U and D_I represent the consumer's demand schedule in, respectively, the absence and presence of perfect information about its quality: they are, respectively, the consumer's 'uninformed' and 'informed' demands[7]. The quantity *actually* purchased is Y_U and this is greater than Y_I, the quantity the consumer *would have* bought had he been fully informed.

The gain in producer's surplus from 'over-consumption' is p_uBAp_I. The loss in consumer's surplus from over-consumption is: Ap_IE – (Op_UBY_U-$OECY_U$) = Ap_IE – (p_UEF – FBC) = Ap_Ip_UF+FBC. So, the net loss to society from 'over-consumption' is loss in consumer's surplus less gain in producer's surplus = Ap_Ip_UF+(AFB+ABC) – (Ap_Ip_UF+AFB)=**ABC**.

When consumers overestimate quality, through a lack of information, producers lack incentives to provide information. Accurate information would lead to a lower surplus for the producer. An analysis, identical to that above, would apply if, due to lack of information, consumers underestimated quality so that there was 'under-consumption'. Now, however, producers would have an incentive to provide information since accurate information would now lead to a higher surplus for the producer.

Commodities for which information is required for satisfactory consumption may be divided into *search* goods and *experience* goods (Nelson, 1970). Information about the attributes of a search good can be determined *prior* to purchase (for example, how comfortable a sofa in a shop is likely to be) whereas information about an experience good can only be obtained *after* purchase (the quality of food in a new restaurant; the reliability of a second-hand car). The effectiveness of an information-gathering strategy depends upon:

(i) the variance in the quality of the good
(ii) the frequency of purchase
(iii) the full price of the good, including any harm from use
(iv) the cost of searching

Search Goods

Search goods may be thought of as a sampling process in which a consumer pays a cost of *$s* to inspect a particular price-quantity combination of a good. The good is rejected if price exceeds the consumer's marginal value for the good. Then the consumer either pays another *$s* to sample another price-quantity combination or stops searching. If the marginal valuation exceeds price the consumer either makes a purchase or continues to search in the hope of finding a more favourable surplus.

The greater the heterogeneity in quality and/or the higher the search costs, the greater the potential for inefficiency through information asymmetry. The point is that search goods rarely involve information asymmetry that lead to significant and persistent inefficiency calling for public policy intervention.

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[7] See Peltzman (1973) for the basic analysis and McGuire, Nelson and Spavins (1975) for a discussion of the empirical problems in using this approach.

Experience Goods

With experience goods, consumers have bear the search cost and the *full price* (*p**) of a good in order to learn about its quality. The full price of a good (Oi, 1973) may be defined as follows. Suppose that a consumer buys Y units at a price of *p* per unit and that the probability of a defective item is *1-q*; then, on average, the consumer expects *Z=Yq* units. If a bad unit inflicts a damage of *W* then the total cost of the purchase (*C*), and the full price (*p**), are defined as:

$$C = pY + W(Y - Z) \text{ and } p^* = \frac{C}{Z} = \frac{p}{q} + W\frac{1-q}{q} \qquad (5)$$

Since the consumer has to purchase the good prior to discovering its quality one would expect that:

(a) sampling would be less frequent, the more expensive the good

(b) sampling would be less frequent, the more durable the good

Furthermore, the consumer may discover, after purchase, that the marginal value is less than price and may regret the purchase.

The Market for 'Lemons': Diagrammatic Analysis

A particular example of 'experience goods' is the market for used cars (Akerlof, 1970). There are two kinds of used cars being sold on the market: 'low-quality' and 'high-quality'. If both sellers and buyers knew whether a given car was low or high quality, there would be a market for low quality cars and a separate market for high quality cars (Figure 11, below).

Source: Pindyck and Rubinfeld (2001), p. 597

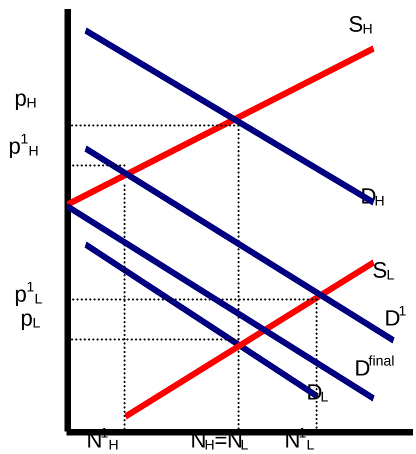

Figure 11
The Market for Lemons

In Figure 11, the price of high-quality cars is p_H and N_H of such cars are sold; the price of low-quality cars is p_L and N_L of such cars are sold. The demand and supply curves of high-quality cars (D_H and S_H) are above the demand and supply curves of low-quality cars (D_L and S_L). The number of high- and low-quality cars is the same (N_H=N_L), but p_H>p_L.

Now suppose that buyers cannot distinguish between high- and low-quality cars. So, if *N* cars were on the market, buyers would regard a given car to be as likely to be a low-quality as a high-quality car. So buyers would be prepared to pay: $p^1 = 0.5p^H + 0.5p^L$ for a car so the new demand curve D^1 lies half-way between D_H and D_L. The price of high-quality cars falls from p_H to p^1_H and the number of high-quality cars sold from N_H to i; and the price of low-quality cars rises from p_L to p^1_L and the number of low-quality cars rises from N_L to N^1_L. This causes buyers to revise downwards the chances of being offered a high-quality car – and to revise upwards the chances of being offered a low-quality car – causing a further leftward shift in the demand curve. The demand curve continues to shift until only low-quality cars are sold (D^{final} in Figure 11).

The Market for 'Lemons': Formal Analysis

Suppose, that the quality of a used car can be indexed by $q,\ q \in [0,1]$. If q is uniformly distributed over the closed interval *[0,1]*, then *E(q)=0.5*. Suppose that there are: a large number of buyers who are prepared to pay a price of aq ($a \geq 1$), and a large number of sellers who willing to accept a price of q, for a car of quality q. If quality was observable, then a car of quality *q* would sell for some price: $p(q) \in (aq, q)$.

But, if quality was not observable, then consumers would estimate the quality of a car by the *average* quality of cars offered on the market. This average quality, denoted $\bar{q}$, can be observed and the consumers' willingness to pay for a car is $a\bar{q}$. Under this circumstance, suppose that the equilibrium price is *p>0*. Then, *only* sellers whose used car is of quality $q \leq p$ will offer their cars for sale, since for the other sellers *p* is less than their reservation price, *q*.

Since quality is uniformly distributed over the interval *[0,p]*, average quality will fall to $\bar{q} = p/2 < 0.5$. Consequently, buyers would only be prepared to pay $a\bar{q} = a(p/2) = (a/2)p < p$ for a car. Hence, no cars would be sold at the price *p*. Since the price *p* was chosen arbitrarily, no used cars will be sold at any positive price *p>0.* Hence, the only equilibrium price is *p=0*, when the demand and supply of used cars is zero: *asymmetric information destroys the market for used cars!* [8]

Adverse Selection

Adverse selection arises when products of different qualities are sold at the same price because, prior to purchase, the buyer cannot distinguish between products of different qualities. Alternatively, adverse selection could arise because a product of uniform quality is sold at the same price to buyers of different qualities and, prior to sale, the seller cannot distinguish between consumers of different qualities[9]. Whatever the sources of adverse selection, the consequence is the same: low-quality products, or high-risk buyers, 'crowd out' high-quality products, or low-risk buyers, so that what is observed is an *adverse selection* of products (as sellers of high-quality products withhold their product) or an *adverse selection* of buyers (as low-risk customers withhold their custom).

Adverse selection represents market failure since 'good' products and 'good' customers are under-represented, and 'bad' products and 'bad' customers are over-represented, in the market. The source of the market failure is the *externality* between products and between customers: when a seller of a low-quality product increases sales, he lowers the average quality of the product on the market, reduces the price the consumer is willing to pay and, thereby, hurts sellers of high-quality products; when a high-risk person buys insurance, he raises the average risk of the contingency; this increases the average

[8] The analysis is from Varian (1992), p. 468.
[9] An example is the insurance industry where the same premium, for a policy against a particular contingency, is charged to different individuals embodying different levels of risk in respect of the insured contingency.

premium the insurance company charges and, thereby, hurts low-risk persons.

Under adverse selection, therefore, sellers of high-quality products will have an incentive to *signal* to the consumer the quality of their product. This may take the form of: *reputation*; *standardisation*; *informative advertising*; offering *warranties* in the event of defects. The signal may be offered through third parties: *recommendations* by friends or by consumer reports; *certification of quality* by a professional association. Educational qualifications, analysed below, are an important way that potential employees signal their worker-qualities to employers.

Education as a Market Signal

A model of the education market is due to Spence (1974). In this model, there are two types of workers: ‘good’ workers and ‘bad’ workers. Good workers have a marginal product of a_G and bad workers have a marginal product of a_B: $a_G > a_B$. A fraction q of the workers are ‘good’, the remainder, $1-q$ are ‘bad’. The production function is linear, so that if L_G good, and L_B bad, workers are employed, output is:

$$Y = a_G L_G + a_B L_B \tag{6}$$

If worker quality was easily observable, the wage paid to each group would equal its marginal product: $w_G = a_G$ and $w_B = a_B$. But if a firm cannot observe worker quality, it offers the average wage to each group:

$$w = q\, a_G + (1-q)\, a_B \tag{7}$$

Now suppose that workers can acquire education and that the cost of acquiring education is lower for good workers than for bad workers: W_G and W_B are the ‘amounts’ of education acquired by good and bad workers and p_G and p_B are the costs of one unit of education for good and bad workers, $p_G < p_B$. Then the total cost of education of good and bad workers is:

$$C_G = p_G \Omega_G \text{ and } C_B = p_B \Omega_B \tag{8}$$

There are now two decisions to be made:

(i) Workers have to decide how much education to acquire

(ii) Firms have to decide how much to pay workers with different levels of education

Assume that education does nothing to increase productivity; its only value is as a signal. Now the firm adopts the following decision rule: for an education level, W^*, pay a wage of a_G if W ³ W^* and pay a wage of a_B if $W < W^*$. In other words, education is taken as an indicator of worker quality and W^* separates workers into good and bad workers.

If under this rule, good workers acquire a level of education W^* or more, and bad workers acquire a level of education less than W^*, then the education level of a worker will perfectly signal his quality. The question is: would it be worthwhile for a bad worker to acquire an education level W^*? The cost of doing so is $p_B W^*$ and the benefit from doing so is the increase in wages: a_G-a_B. So a bad worker will not acquire W^* education if:

$$p_B \Omega^* > a_G - a_B \quad (9)$$

and a good worker will acquire W^* education if:

$$p_G \Omega^* < a_G - a_B \quad (10)$$

So provided W^* satisfied the condition:

$$\frac{a_G - a_B}{p_B} < \Omega^* < \frac{a_G - a_B}{p_G} \quad (11)$$

the education of a worker will perfectly signal his quality. This type of equilibrium is called a *separating equilibrium* since it allows each type of worker to make a choice which separates him from the other type.

If, however, $p_B \Omega^* < a_G - a_B$ bad workers will also acquire the education level W^* and if $p_G \Omega^* > a_G - a_B$ even good workers will not acquire any education. So $\Omega^* < \frac{a_G - a_B}{p_B}$ or $\Omega^* > \frac{a_G - a_B}{p_G}$ will lead to a *pooling equilibrium* in which both types of workers make the same choice and the firm has to pay the average wage *w* of equation (7).

The separating equilibrium is socially inefficient because each good worker pays to acquire the education level W^*, even though it does nothing to increase his productivity, simply to distinguish himself from a bad worker. Exactly the same output is produced with signalling as without signalling (equation (6)), it is just that the distribution of rewards is different. So, under the terms of the model, investment in education confers a private gain (to the good workers who can earn more than bad workers) but no social benefit.

6. Principal-Agent Issues

Adverse selection arises because one side of the market cannot observe the quality of the product or the customer on the other side: consequently, it is sometimes referred to as a *hidden information* problem. A related issue is the fact that the actions of a person in transaction may not be observable by the other party (parties) to the transaction. This may then lead the party, whose action cannot be observed, to be 'negligent', or to not take 'due care', in fulfilling his duties with respect to the transaction. This is referred to as *moral hazard* or a *hidden action* problem and it underpins issues of *principal-agent* interaction.

Full Information

The analysis[10] first assumes that there is no hidden action problem: the action of the agent can be observed by the principal. A firm is hiring a worker to do a job. The output of the worker, y, depends amount of 'effort', x, that the worker puts in: $y=y(x)$; the payment to the worker, s, depends upon the output produced: $s=s(y)$. The firm would like to choose the payment function, $s(y)$, so as to maximise its surplus: $y - s(y)$.

However, the worker finds effort to be costly and $c=c(x)$ denotes the cost of effort. The utility of a worker who chooses effort x is: $s(y(x)) - c(x)$. If $\overline{u}$ is his 'reservation' utility level, then the *participation constraint* is:

$$s(y(x)) - c(x) \geq \overline{u} \tag{12}$$

[10] Varian (2003), p. 679.

Given this constraint, the firm wants the worker to choose *x* so as to maximise surplus:

$$\underset{x}{Max}\ y(x) - s(y(x)) \text{ st } s(y(x)) - c(x) = \overline{u} \qquad (13)$$

It is assumed that the firm can perfectly observe the effort of the worker so there is no asymmetry of information.

Substituting the constraint into the objective function in equation (13) yields:

$$\underset{x}{Max}\ y(x) - [c(x) + \overline{u}] \qquad (14)$$

and the first order conditions for this are:

$$y'(x) = c'(x) \qquad (15)$$

or the marginal product from extra effort = marginal cost of extra effort.

Suppose x^* is the optimal level of effort which the firm wants to extract from the worker. Then the incentive scheme *s(y)* must be such that x^* gives the worker the highest level of utility:

$$s(y(x^*)) - c(x^*) \geq s(y(x)) - c(x)\ \ \forall x \qquad (16)$$

The constraint embodied in equation (16) is the *incentive compatibility constraint*.

So, the incentive scheme *s(y)* must satisfy the participation constraint (equation (12)) and the incentive compatibility constraint (equation (16)).

Rent The worker could pay a fixed amount, call it 'rent' (*R*), to the firm and keep the remainder. Then the payment scheme is:

$$s(y(x)) = y(x) - R \qquad (17)$$

If the worker chooses effort so as to maximises his utility, *s(y(x))-c(x)*, then under 'rent' his problem is:

$$\underset{x}{Max}\ y(x) - c(x) - R \qquad (18)$$

and the first order conditions for this are: $y'(x) = c'(x)$ so that the worker supplies the optimal level of effort x^*. The optimal rent, R^*, is that which is exactly high enough (equation (12)) to induce the worker to participate:

$$R^* = y(x) - c(x) - \overline{u} \tag{19}$$

Wages Now the worker is paid a wage, *w*, per unit of effort plus a lump sum, *K*, so that the payment scheme is:

$$s(x) = wx + K \tag{20}$$

The worker then chooses effort so as to maximise:

$$wx + K - c(x) \tag{21}$$

and the first order conditions for this are $w = c'(x)$. So, if the wage paid is equal to the worker's marginal product, he will choose the optimal level of effort given by equation (15). The payment *K* is chosen so that: $K = \overline{u} + c(x) - c'(x)x$.

Take-it-or-leave-it Under this scheme, the worker receives a payment K^* for an effort level of x^*, nothing otherwise, where K^* satisfies the participation constraint: $K^* = \overline{u} + c(x^*)$. If $x \neq x^*$, then he receives utility $-c(x)$, but if $x=x^*$, utility is $\overline{u}$: hence $x=x^*$, is the optimal choice.

Sharecropping Under sharecropping, the worker and the firm each receive a fixed share of output (respectively, a and $1-a$). Suppose the worker receives: $s(x) = a\, y(x) + K$, where *K* is a fixed payment and $a<1$. Then the worker chooses *x* to maximise:

$$a\, y(x) + K - c(x) \tag{22}$$

and, at an optimum: $a\, y'(x) = c'(x)$. This violates the condition for optimum effort of equation (15): $y'(x) = c'(x)$.

Under all the schemes, the worker chooses *x* so as to maximising his utility: *s(y(x))-c(x).* This implies that he chooses his effort by equating *the marginal benefit of effort with its cost*. The firm wants him to choose his effort so as to equate *the marginal product of effort with its cost*. This can be achieved if the payment scheme provides a *marginal benefit to the worker which is equal to his marginal product*. All such schemes are incentive-compatible (Rent; Wages; Take-it-or-leave-it) and, therefore, *efficient*; any scheme that does not provide a marginal benefit equal to marginal product is not incentive-compatible and, therefore, *inefficient*.

Hidden Action: Efficiency Wages

If the effort that the worker puts in is not observable by the firm then it has two alternatives: the firm can motivate the worker by providing a wage higher than the market clearing wage; the firm can monitor the effort of the worker and threaten to fire those whose effort is unacceptably low. The higher the current rate of unemployment, and the higher the wage paid over the market wage, the more effective will be the threat of dismissal (Yellen, 1984).

Source: Pindyck and Rubenfeld (2001), p. 618

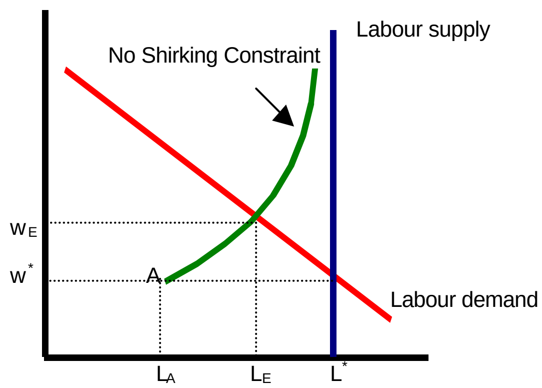

Figure 12:
Efficiency Wages

In Figure 12, the market clearing wage w^* equates labour demand with labour supply. However, when workers are paid w^*, they have an incentive to 'shirk': even if they are detected and fired, they can find a job with another employer for the same wage because, by definition, there is no unemployment. So the firm has to offer an 'efficiency wage' $w_E > w^*$ so that there is 'no shirking'. This efficiency wage is shown by the 'no-shirking constraint' curve in Figure 12.

This curve shows, for each level of labour usage, the minimum wage that workers need to be paid so that they do not shirk. At the point A, labour usage is L_A and unemployment is L^*-L_A: workers only need to be paid the competitive wage, w^*, to induce them not to shirk; as labour usage rises, so that the unemployment rate falls, the efficiency wage rises above the competitive wage. The equilibrium efficiency wage – which all firms in the industry pay – is w_E, given by the intersection of the labour demand and the 'no shirking' curve. Unemployment is L^*-L_E and workers do not shirk at w_E because the positive amount of unemployment at w_E means they would not be certain of finding another job if they were fired for shirking.

7. Public Goods

Private goods are *rivalrous* (a unit of a good consumed by one person cannot also be consumed by another person) and *excludable* (a person who does not pay for a good can be excluded from its consumption) in consumption. By contrast, a **pure public good** is *non-rivalrous* (a given amount of the good can be consumed by one person without affecting its simultaneous consumption by another) and *non-excludable* (non-payment does not entail exclusion from consumption). Within these poles of non-rivalrousness and non-excludability, **impure public goods** represent in between cases. Impure public goods arise because of *congestion costs*: the value to existing users of a public good falls as more users are added. Impure public goods are, therefore, *partially rivalrous*. Within this category of impure public goods, it is possible to distinguish between:

(i) **Common Property Resources:** public goods subject to congestion from which exclusion is not possible

(ii) **Club goods:** public goods subject to congestion from which exclusion is possible

(i) **Variable Use goods:** public goods subject to congestion where the amount of services used by consumers can be varied

The Efficient Provision of a Discrete (Pure) Public Good

There are two goods - a 'private' good and a 'public' good – and two persons, indexed *i=1,2*. The public good is either provided (*G=1*) or not provided (*G=0*) and the cost of providing it is *C*; the private good, on the other hand, is supplied in varying quantities. W_i is the wealth of the agent *i* and X_i is his expenditure on the private good and G_i is his contribution towards the provision of the public good. Consequently:

$$G\begin{cases} =0 \text{ if } G_1+G_2<C \\ =1 \text{ if } G_1+G_2\geq C \end{cases} \qquad (23)$$

If $U_1(X_1, G)$ and $U_2(X_2, G)$ are the utility functions of the two persons, then it will be efficient to provide the public good[11] (*G=1*) if for some pattern of contributions (G_1, G_2), such that G_1+G_2 ³ C:

$$U_i(W_i - G_i, 1) \geq U_i(W_i, 0) \;\; \forall i = 1,2 \tag{24}$$

with the strict inequality, >, holding for at least one *i*.

Define the reservation price of consumer *i* for the public good (which is the consumer's maximum willingness-to-pay for the public good) as R_i where:

$$U_i(W_i - R_i, 1) = U_i(W_i, 0) \tag{25}$$

Then a necessary and sufficient conditions for the provision of the public good to be Pareto improving are:

$$R_i > G_i, i = 1,2 \text{ and } \sum_i^2 R_i > \sum_i^2 G_i \geq C \tag{26}$$

How effective would the market be at providing the public good? Suppose that $R_i=100$, $i=1,2$ and $C=150$ so that by equation (26), it is efficient to provide the public good.

		Consumer 2	
		Buy	Don't Buy
Consumer 1	Buy	-50,-50	-50,100
	Don't Buy	100,-50	0,0

Source: Varian (1992), p. 417.

Each consumer has to decide independently whether or not to buy the public good: if consumer 1 buys, then 2 can *free ride* by not buying; similarly, if consumer 2 buys, then 1 can *free ride* by not buying. Therefore, as in the above table of payoffs, the dominant strategy is for both consumers to *not buy* the public good[12]. So the net result is that the good is not provided even though it would be efficient to do so.

The Efficient Provision of the Quantity of a (Pure) Public Good

When the quantity of the public good can be varied, let G denote the quantity of the public good (where *G=0* implies no provision) and let *C=C(G)* denote

[11] Or, conversely, it will be inefficient to not provide the public good (G=0).
[12] This game has a structure similar to that of the *Prisoner's Dilemma*.

the cost of provision. Then an efficient combination of the amounts of the private and the public good will be produced when one consumer's utility is maximised, subject to the other consumer's utility being fixed at some level and subject to the budget constraint[13]:

$$\max_{X_1, X_2, G} U_1(X_1, G) \text{ s.t. } U_2(X_2, G) = \bar{U}_2 \text{ and } X_1 + X_2 + C(G) = W_1 + W_2 \qquad (27)$$

This yields the equilibrium condition:

$$\frac{\partial U_1(X_1, G)/\partial G}{\partial U_1(X_1, G)/\partial X_1} + \frac{\partial U_2(X_2, G)/\partial G}{\partial U_2(X_2, G)/\partial X_2} = C'(G) \qquad (28)$$

or, in other words:

$$MRS^1_{XG} + MRS^2_{XG} = MC_G \qquad (29)$$

The equilibrium condition in equation (29) is known as the *Samuelson Condition* (Samuelson, 1954) for the efficient provision of a (pure) public good and is illustrated in Figure 13, below. (This condition is derived in the Mathematical Appendix). For two consumers, A and B, the optimal production is O_AX_0 of the private good and O_AG_0 of the public good. The optimal distribution of the private good (so that A is kept on the indifference curve II) is O_AZ to A and O_AB to B. At the point of tangency between TT and JJ we have: $MRS^B_{GX} + MRS^A_{GX} = MRT_{GX}$.

Example. Suppose C(G)=G and the utility functions are Cobb-Douglas so that $U_i(X_i, G) = \log X_i + b_i \log G$. Then $MRS^i_{XG} = (\partial U_i / \partial G)/(\partial U_i / \partial X_i) = b_i X_i / G$ and the efficiency condition is: $\sum b_i X_i / G = 1 \Rightarrow G = \sum b_i X_i$. Using the constraint $\sum X_i + G = W$ defines the set of efficient allocations: $(G, X_1 .. X_N)$. Note that there may be many levels of efficient provision of the public good.

Example: Suppose C(G)=G and the utility functions are quasi-linear[14] so that $U_i(X_i, G) = X_i + b_i \log G$. Then $MRS^i_{XG} = (\partial U_i / \partial G)/(\partial U_i / \partial X_i) = b_i / G$ and the efficiency condition is: $\sum b_i / G = 1 \Rightarrow G = \sum b_i$ so that there is a *unique* efficient level of provision of the public good.

[13] See Varian (2003), p. 666.
[14] This implies that the marginal utility of the private good is always 1.

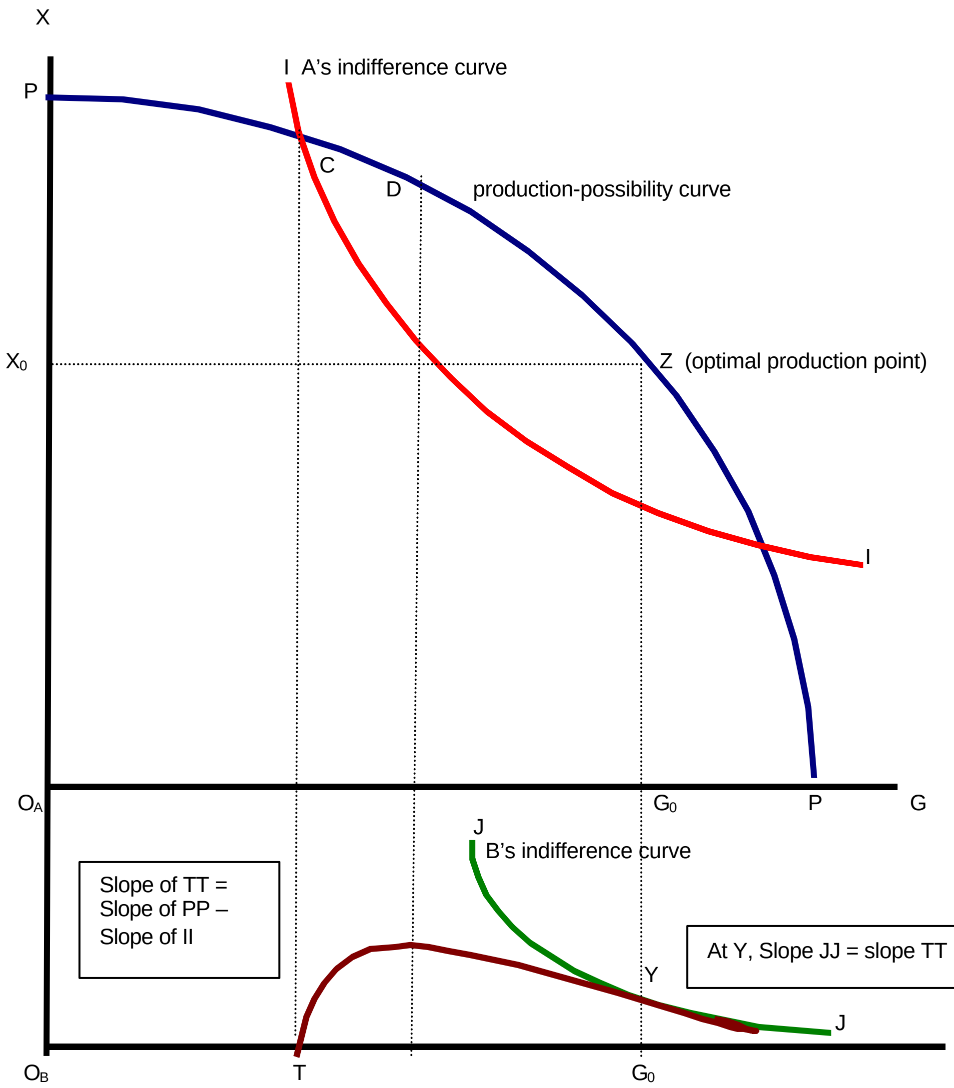

Figure 13:
The Samuelson Condition for the Efficient Provision of a Public Good

Free-Riding and the Private Provision of a Public Good

Suppose consumers 1 and 2 are independently deciding their contributions (G_1 and G_2) to the public good: $G_1+G_2=G$; $C(G)=G$. So if 1 thinks that 2 will contribute G_2, his problem is[15]:

$$\underset{G_1}{Max}\ \ U_1(W_1 - G_1, G_1 + G_2) \text{ s.t. } G_1 \geq 0 \tag{30}$$

The Kuhn-Tucker first-order conditions for solving this are:

$$\frac{\partial U_1(W_1 - G_1, G_1 + G_2)}{\partial G} - \frac{\partial U_1(W_1 - G_1, G_1 + G_2)}{\partial X_1} \leq 0 \tag{31}$$

which may be written as:

$$MRS^1_{XG} = \frac{\dfrac{\partial U_1(W_1 - G_1, G_1 + G_2)}{\partial G}}{\dfrac{\partial U_1(W_1 - G_1, G_1 + G_2)}{\partial X_1}} \leq 1 \tag{32}$$

where equality in equation (33) holds if $G_1>0$. So, if consumer 1 contributes a positive amount to the public good ($G_1>0$), he will equate his marginal rate of substitution between the private and public good to the marginal cost of providing the public good[16]. If $MRS^1_{XG} < 1$, then he will not want to contribute ($G_1=0$) (Bergstrom, Blume and Varian, 1986).

In Figure 14, below, the initial endowment of consumer 1 is at point A (G_2, W_1) In left-hand panel he contributes an amount $G_1>0$ to the public good and moves to point B. In the right-hand panel, he 'free rides' on consumer 2's contribution of G_2 to the public good and remains at A ($G_1=0$).

Example: Suppose C(G)=G and the utility functions are (quasi-linear): $U_i(X_i, G) = X_i + b_i \log G$. Then the equilibrium conditions are: $(b_1/G) \leq 1$, $(b_2/G) \leq 1$.

In general, only one of the constraints can be binding: if $\beta_2>\beta_1$, only consumer 2 will contribute and 1 will free ride. Only when $\beta_2=\beta_1$, will both contribute.

[15] Note that equation (31) incorporates the constraint: $X_1 + G_1 = W_1$

[16] Note that since, by assumption, C(G)=G, $MC_G=1$.

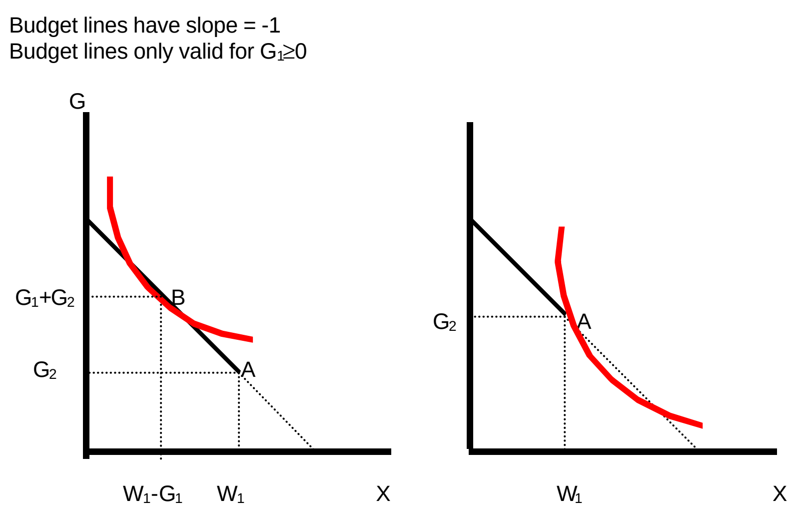

Figure 14:
Private Provision of a Public Good

Rewrite the consumer's optimisation problem of equation (31) as:

$$\underset{X_1, G}{Max}\ U_1(X_1, G) \text{ s.t. } G \geq G_2 \text{ and } G + X_1 = W_1 + G_2 \quad (33)$$

Solving this problem yields the consumer 1's demand function for the public good as: $G = f_1(W_1 + G_2)$. Then the amount of the public good is:

$$G = Max\{f_1(W_1+G_2), G_2\} \Rightarrow G_1 = Max\{f_1(W_1+G_2) - G_2, 0\} \quad (34)$$

Equation (35), defines the reaction function for consumer 1 by giving his optimal contribution as a function of the contribution of consumer 2. A *Nash Equilibrium* is a set of contributions, G_1^*, G_2^* such that:

$$\begin{aligned} G_1^* &= Max\{f_1(W_1 + G_2^*) - G_2^*, 0\} \\ G_2^* &= Max\{f_2(W_2 + G_1^*) - G_1^*, 0\} \end{aligned} \quad (35)$$

Lindahl Pricing

Under *Lindahl pricing*, every consumer *i* is charged a price p_i for the public good and is offered the right to buy as much of the public good as he wishes at the price. Therefore, the maximisation problem for consumer *i* is:

$$Max\ U_i(X_i, G) \text{ s.t. } X_i + p_i G = W_i \quad (36)$$

and the first order condition for solving this problem is:

$$\frac{\partial U_i(X_i, G)/\partial G}{\partial U_i(X_i, G)/\partial X_i} = p_i \quad (37)$$

The optimal amount of the public good demanded by consumer *i* is: $G_i^* = G_i(W_i, p_i)$.

The question is does there exist a set of prices $p_i^*, i = 1..N$, such that consumers will all choose an efficient amount of the public good: $G^* = G_1^* = G_2^* = .. = G_N^*$. By equation (29), an efficient amount of the public good must satisfy: $\sum_{i=1}^{N} \frac{\partial U_i(X_i^*, G^*)/\partial G}{\partial U_i(X_i^*, G^*)/\partial X_i} = C'(G^*)$ and so setting prices such that:

$$p_i^* = \frac{\partial U_i(X_i^*, G^*)/\partial G}{\partial U_i(X_i^*, G^*)/\partial X_i} \quad (38)$$

will support an efficient amount of the public good. These prices - which are set equal to each consumer's marginal rate of substitution between the private and the public good – are known as *Lindahl prices* (Lindahl, 1919). These prices may also be interpreted as tax rates.

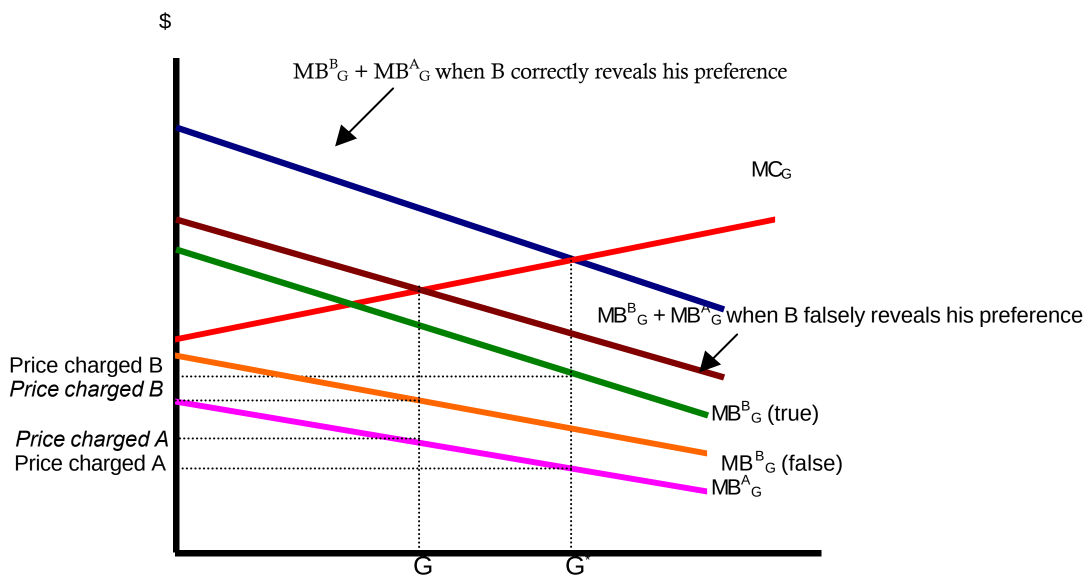

Figure 15:
Lindahl Prices and Under-Revelation of Preferences

In Figure 15, above, the optimal level of the public good is G*, when $MRS^1_{XG} + MRS^2_{XG} = MC_G$. At this level of provision, A and B pay, respectively, p_A and p_B, p_A<p_B. In order to avoid paying p_B, B under-declares his preference. As a consequence, provision of the public good falls to *G*, below the optimal level of provision, *G**. So, Lindahl pricing could lead to under-provision of the public good if consumers falsify their preferences in order to reduce the prices (tax rates) they have to pay.

Lindahl pricing could also lead to under-provision of a public good if some consumers free ride on the demand of another consumer. In Figure 16, below, consumer A, free rides on B's demand for the public good: provision is at G_B, instead of at G^*, causing a net social loss equal to the area of the triangle XYZ.

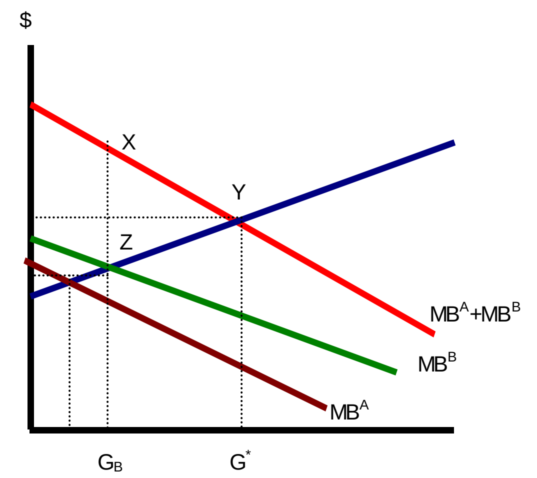

Figure 16:
Free riding Under Lindahl Pricing

Common Property Resources: Congestion but Non-Excludability

Consumption of a "common property" resource is rivalrous in the sense that each additional user of the resource reduces the return to the existing users. Hence there are social as well as private costs to adding new users. However, because all users (existing as well as potential) have free access to the resource, no one can be excluded from using the resource. Hence it is a "common property" or an "open access" resource.

Suppose that a community has access to a lake for fishing and that the lake has a limited number of fish which means that the more the number of fisherman that use the lake, the fewer the fish that each fisherman catches. This is shown in the table below.

Fishermen	*Fish per Fisherman*	*Total Catch*	*Marginal Catch*
1	10	10	10
2	9	18	8
3	8	24	6
4	7	28	4
5	6	30	2
6	5	30	0

Source: Liebowitz and Margolis (1999), p. 74

Suppose that the alternative wage for the fishermen is another occupation which pays (the equivalent of) six fish. Then, under open access, 5 fishermen will fish on the lake. But the *tragedy of the commons* is that the fourth and fifth fishermen will only generate, respectively, four and two additional fish. *From a social perspective*, their energies would have been better spent elsewhere, in the alternative occupation. But, since they only look to the average catch and not to the marginal catch, the lake is over-fished by two fishermen. This is depicted in Figure 17, below. The loss due to over-fishing is the area of the triangle, *ABC*.

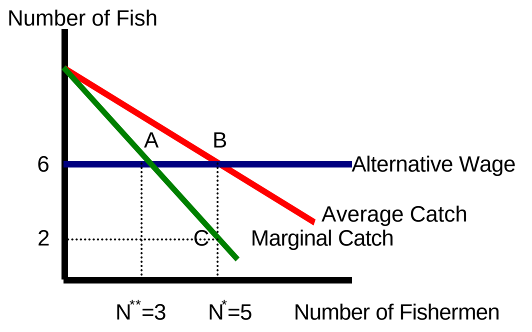

Figure 17:
The Tragedy of the Commons

Suppose it costs *$z* to 'equip' a fisherman. The total number of fish caught depends on how many fishermen are already there: let *f(N)* represent the number of fish caught, if there are N fishermen, so that $f(N)/N$ is the average catch. To maximise the *social surplus* choose N so as to maximise:

$$\underset{N}{Max}\ f(N) - zN \tag{39}$$

and the first-order condition for this is:

$$f'(N) = z \tag{40}$$

Each potential fisherman, however, will compare the average catch, after he has joined, with the cost of fishing: if $f(N+1)/(N+1)>z$ he will fish, otherwise he will not. But since $f(N+1)/(N+1)>f'(N)=f(N+1)-f(N)$, over-fishing will result[17].

Club Goods: Congestion and Excludability

The congestion property implies that as additional users are added the benefits that previous users obtained from a given quantity of the public good declines. In Figure 18, below, the curve TB shows how the benefits per user falls, while the curve AC shows how the cost-per-user falls, as the number of users increases. The slope of TB is the marginal benefit, and the slope of AC is the marginal cost-per-user, from more users[18]. The optimal number of users occurs when marginal benefit = marginal cost (Buchanan, 1965).

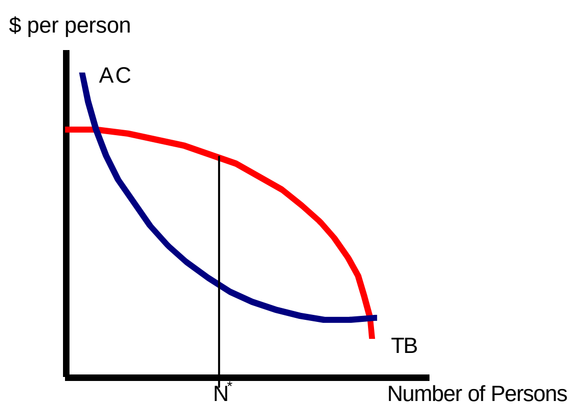

Figure 18:
The Optimal Number of Users of a Club Good

[17] When the average is falling, the marginal lies below the average: $d(f(N)/N)/dN < 0 \Rightarrow df(N)/dN < f(N)/N$

[18] $MC = d(C/N)/dN = -C/N^2$

Variable-Use Public Goods

Suppose there are *N* cars using a bridge of capacity *K* with an average crossing time of *T(N,K)*: the total time in crossing the bridge is $N \times T(N,K)$. An additional car increases congestion on the bridge and the average crossing time increases to *dT(N,K)/dN*. So, the total time in crossing the bridge increases by $N \times (dT(N,K)/dN)$. This increase is the *social marginal cost* imposed by the additional traveller.

In Figure 19, below, the optimal number of cars crossing the bridge is N^{**} but actually N^{*} cars use the bridge (at N_0, there are no congestion costs). This is because new users do not take account of the additional cost they impose on all the existing users. Charging a toll on the bridge removes the excess usage.

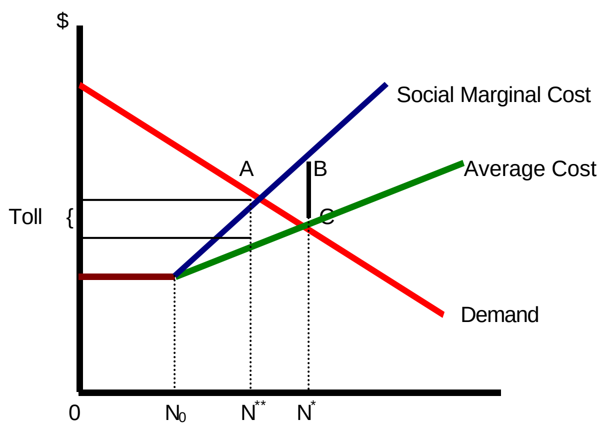

Figure 19
Congestion Costs

8. Externalities

An *externality* exists when the action of one agent unavoidably affects the welfare of another agent. The affected agent may be a consumer, giving rise to a *consumption externality*, or a producer, giving rise to a *production externality*.

Suppose there are two firms. Firm produces output *y* at a cost *C(y)* and imposes an externality in the form of a cost *E(y)* on firm 2. If *p* is the price of firm 1's output, profits for the firms are:

$$p_1 = py - C(y) \text{ and } p_2 = -E(y) \qquad (41)$$

Firm 1 chooses output by maximising p_1 and the profit maximising level of output occurs when: $p = C'(y)$. However, since Firm 1 takes account only of private costs, the profit-maximising output, y^* is, from a social perspective, too high. If the two firms merged in order to *internalise* the externality, the merged firm would maximise: $p = py - C(y) - E(y)$ and the profit maximising output would occur when: $p = C'(y) + E'(y)$ that is when price was equal to marginal social cost. In Figure 20, below, the social cost of the externality is the area of the triangle, *ABC*.

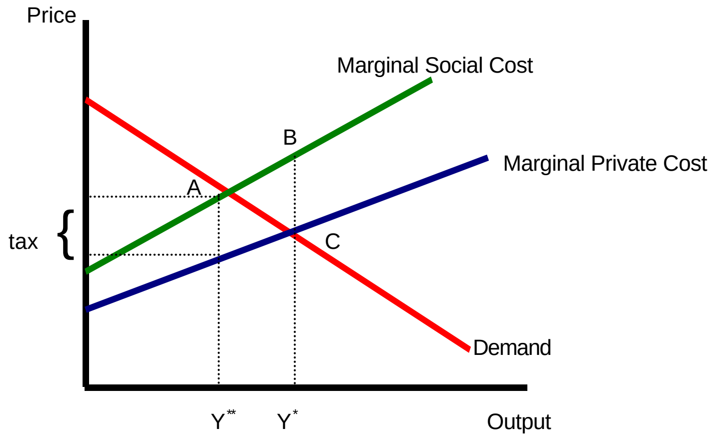

Figure 20:
The Social Cost of An Externality

Solutions to Externalities

A. Pigovian Taxes.

The externality problem arises because Firm 1 faces the 'wrong' prices and these lead it to produce at Y^* instead of at Y^{**}. A corrective tax (first proposed by Pigou, 1920) can be imposed which, by yielding the 'right' prices, will induce it to produce at Y^{**}. If *t* is the tax rate per unit of output, then the firms post-tax marginal cost is: $C'(y) + t$. If the tax rate is set so that: $t = E'(y)$, the socially desirable output will be produced. The problem with this solution is that it requires the taxing authority to know the externality damage function, $E(y)$ and, hence, the optimal level of output, Y^{**}. But, if this was known, the firm could simply be regulated to produce Y^{**}.

B. Missing Markets

The reason that the 'externality problem' arises is that there is no market for the output of the externality. The producer of an externality-generating product does not have to bear any cost for producing the concomitant externality output. The problem is, therefore, one of *missing markets* and *ipso facto* may be solved by creating a market for the output of the externality.

Suppose when y_1 units of output are produced by firm 1, $z=f(y_1)$ units of the 'nuisance commodity' are unavoidably produced. Suppose firm 2 has the right to be free of the nuisance and let q be the price of the nuisance good. Then the optimising problem facing firms 1 and 2 are:

$$\begin{aligned} &\underset{y_1}{Max}\ \pi_1 = p_1 y_1 - C(y_1) - qf(y_1) \\ &\underset{y_2}{Max}\ \pi_2 = p_2 y_2 + qf(y_1) - C(y_2) - E(f(y_1)) \end{aligned} \qquad (42)$$

where p_i and y_i are, respectively, price and output of firm i. The first-order conditions for solving this optimisation problem are:

$$\begin{aligned} p_1 &= C'(y_1) + qf'(y_1) \\ p_2 &= C'(y_2) - qf'(y_1) + E'(z)f'(y_1) \end{aligned} \qquad (43)$$

Then setting the price of the nuisance good to its marginal cost ($q = E'(z)$) in equation (43) rids firm 2 of the adverse effect of the externality on its costs and forces firm 1 to bear the cost of producing the nuisance good associated with the externality.

C. Property Rights

This solution is due to Coase (1960) and is often referred to as the 'Coase Theorem'. This theorem says that when parties can bargain to their mutual advantage *without cost*, then the resulting outcome will be efficient, *regardless of how property rights are distributed*. Put differently, the 'Coasian solution' to the problem of externalities is to establish institutions which will define and enforce property rights and which will allow parties to bargain at zero transaction cost.

Coase's first point was that externalities are the joint product of the 'offender' and the 'victim' *and the most efficient system of avoiding an externality is to put the onus for avoidance on the party which can avoid it at the least-cost*. The Pigovian solution of penalising the generator of externalities would only be efficient if this party was the lowest cost avoider; otherwise, it would be inefficient.

Coase's second point was that in order to remove the ill-effect of an externality, neither regulation nor taxes were necessary. If transaction costs were zero – that is, any agreement that is in the mutual interest of the parties can be arrived at costlessly – then bargaining between the parties would lead to an efficient outcome, regardless of how property rights were defined.

Coase's third point was that the problem was not one of externalities but, rather, one of transaction costs which prevented externalities being bargained out of existence. When we observed externalities in the real world, Coase would have us enquire about the level of transaction costs which prevented externalities being bargained away.

9. Risk and Uncertainty

The problem of risk and uncertainty arises when goods are distinguished by 'time of consumption' and by 'state of nature' (or *contingency*). The assumption that efficient markets exist for goods under all contingencies, and at all times, assumes that consumers can buy insurance on actuarially fair terms so that, regardless of contingency, utility remains unchanged.

The Demand For Insurance

Suppose a consumer has a wealth of \$W and there is some probability π that he will lose *\$X* in an adverse contingency. The consumer can buy insurance that will pay him *\$Z* in the event of this contingency and the price of insurance is γ per $ of cover. To find out how much insurance the consumer will purchase, formulate his problem as one of maximising his *expected utility*:

$$\underset{Z}{Max}\ EU(Z) = p\,U(W - X - g\,Z + Z) + (1-p)U(W - g\,Z) \tag{44}$$

The first order condition for solving this problem is:

$$EU'(Z) = p\,U'[W - X - (1-g)Z](1-g) - (1-p)U'[W - g\,Z]g = 0 \tag{45}$$

which simplifies to:

$$\frac{p}{1-p}\frac{U'[W - X - (1-g)Z]}{U'[W - g\,Z]} = \frac{g}{1-g} \tag{46}$$

If the insurance company charged an actuarially fair premium (γ=π), the consumer would be fully insured since:

$$\begin{aligned} &U'[W - X - (1-g)Z] = U'[W - g\,Z] \\ &\Rightarrow W - X - (1-g)Z = W - g\,Z \Rightarrow X = Z \end{aligned} \tag{47}$$

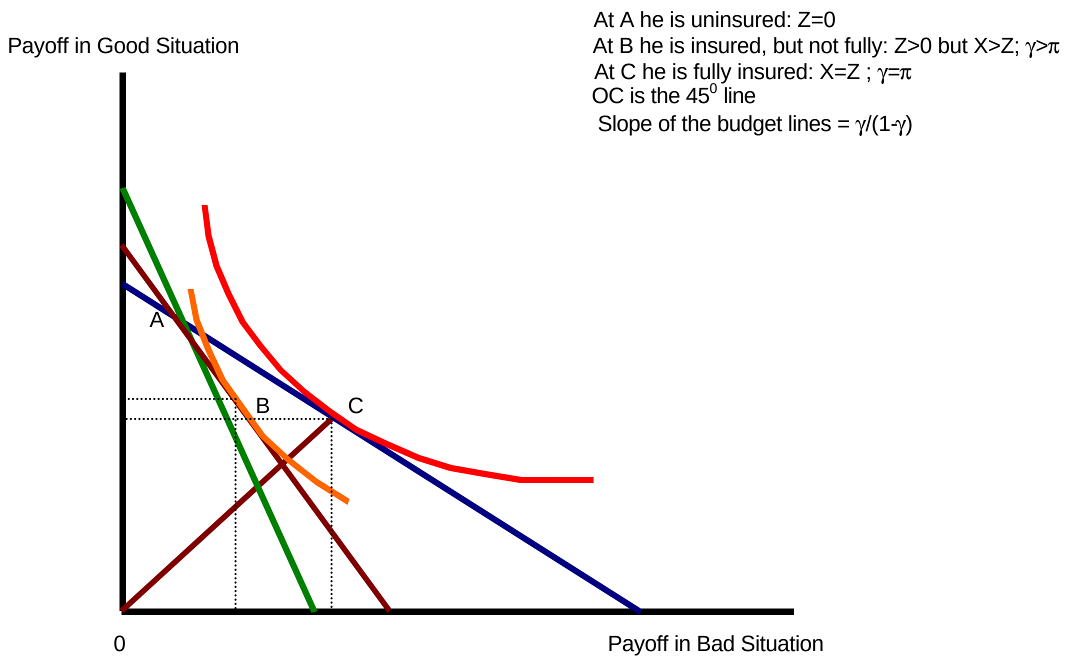

Figure 21:
The Demand for Insurance

The expected profit of the insurance company is:

$$\begin{aligned} & p(gZ-Z)+(1-p)gZ \\ & =(g-p)Z \end{aligned} \tag{48}$$

and so, if the insurance industry is competitive, profits would be competed away and the insurance company would charge an actuarially fair premium ($\gamma=\pi$). Conversely, if barriers to entry meant that there competition in the insurance industry was imperfect, then insurance firms would continue to make supernormal profits (with $\gamma > \pi$) and there would be 'market failure' due to the fact that consumers would not be fully insured (see Figure 21, above).

Investment in a Risky Asset

Suppose a consumer has an initial wealth of *$W* and is considering investing an amount *$X* of this in a risky asset. This asset could earn $r_B < 0$ in a bad outcome (which occurs with probability p) and $r_G > 0$ in a good outcome (which occurs with probability $1-p$)[19]. Then the consumer's wealth in the good and bad outcomes is:

$$\begin{aligned} W_G &= (W-X)+X(1+r_G) = W + Xr_G \\ W_B &= (W-X)+X(1+r_B) = W + Xr_B \end{aligned} \tag{49}$$

Then the expected utility of the consumer who invests *$X* is:

$$EU(X) = pU(W+Xr_B)+(1-p)U(W+Xr_G) \tag{50}$$

and the consumer chooses X so as to maximise expected utility. The first-order condition is:

$$\begin{aligned} & EU'(X) = pU'(W+Xr_B)r_B+(1-p)U'(W+Xr_G)r_G \\ & = pU'(W+Xr_B)r_B+(1-p)U'(W+Xr_G)r_G \\ & +pU'(W+Xr_G)r_B - pU'(W+Xr_G)r_B \\ & = U'(W+Xr_G)[pr_B+(1-p)r_G]+pr_B[U'(W+Xr_B)-U'(W+Xr_G)] = 0 \end{aligned} \tag{51}$$

and the second-order condition is:

$$EU''(X) = pU''(W+Xr_B)r_B^2+(1-p)U''(W+Xr_G)r_G^2 \tag{52}$$

If the consumer is risk-averse, the utility function *U(.)* will be concave implying $U''(W) < 0 \forall W$. Consequently, $EU''(X) < 0$ and expected utility will be a concave function of X.

Equation (52) implies that for the first dollar invested (that is, *X=0*):

$$EU'(0) = U'(W)[p\, r_B + (1-p\,)r_G\,] \qquad (53)$$

where the term in brackets in equation (53) is the *expected return* (ER) from the investment. If *ER £ 0*, $EU'(0) \leq 0$ and by risk aversion, $EU'(X) < 0$ for all *X>0*. That is, a risk averse investor will not invest in a risky asset if its expected return is not positive.

The first order condition of equation (51) may be rewritten as:

$$\frac{U'(W + Xr_B)}{U'(W + Xr_G)} = -\frac{r_G}{r_B}\frac{(1-p\,)}{p} \qquad (54)$$

so that in equilibrium, the ratio of the marginal utilities is equal to the ratio of the expected returns.

10. Social Welfare and Inequality

Market failure may also arise because inequality in the distribution of commodities between consumers may mean that the social welfare associated with a given level of production is *sub-optimal*.

The Impossibility Theorem

The basic problem of social welfare is to be able to arrive at a social ranking of the different alternatives which is an ‘accurate’ reflection of the ranking of these alternatives by the individuals in society. One way of deducing the social ranking is by allowing the individuals to vote. For example, every individual in society may rank different 'projects' according to the net benefits that they expect to obtain. The problem is that such a ranking by individuals may not lead to a social ranking, that is to a ranking to which all individuals in society would subscribe. For example with three individuals (A, B and C) and three projects (X, Y and Z) suppose the rankings are as given in the table below:

Preference Ordering	*A*	*B*	*C*
First Choice	X	Z	Y
Second Choice	Y	X	Z

[19] The value of the asset increases in the good state (r_G>0) and decreases in the bad state (r_B<0).

Third Choice	Z Y X

Then in a sequence of pair-wise comparisons: X versus Y, Y wins since both A and B prefer X to Y; Y versus Z, Y wins, since both A and C prefer Y to Z; X versus Z, Z wins since both B and C prefer Z to X. The implied social ordering is that X is preferred to Y; Y is preferred to Z; but Z is preferred to X! The cyclical nature of social preferences arises from the fact that the social ordering is not transitive or, in the language of electoral studies, there is no *Condorcet winner*. Indeed, the problem of social choice is not unlike that of voting behaviour: in both cases the issue is one of translating individual preferences into an agenda for collective action that faithfully represents these preferences.

More generally, the possibility of intransitivity in social rankings – of the sort described above – is not necessarily the result of obtaining such rankings from pair-wise majority rule voting; intransitivity can occur from the application of any rule for creating social rankings which satisfies certain minimal properties. This was demonstrated by Arrow (1951), in his celebrated 'Impossibility Theorem', when he showed that any social rule which satisfied a minimal set of fairness conditions[20] could produce an intransitive ranking when two or more persons had to choose from three or more projects.

Arrow's result rendered all democratic rules of collective action suspect - the idea that the state could act in terms of a well-defined social interest by aggregating over individual preferences (Bergson, 1938) was now rendered invalid. The work of Black (1948) and Arrow (1951) work also drew attention to the potentially unstable nature of majority coalitions. Although the problem of cyclical voting had been known of since Condorcet, Black's and Arrow's work brought out its relevance to policy analysis. Variations and extensions of

[20] These conditions were the axioms of: *unrestricted domain* (individuals had transitive preferences over all the policy alternatives); *Pareto choice* (if one project made someone better off than another project, without making anyone worse off, then it would be the socially

Arrow's (1951) result have taken the form of investigating whether the theorem would continue to be true when one or the other of these axioms was weakened.

The Social Welfare Function

One property that may be dropped from Arrow's list of desirable properties (see footnote) is the requirement that the social preferences between two alternatives depends *only* on the individual ranking of these alternatives. Define for individual *i*, the utility associated with alternative *X* as $U_i(X)$ and define the social welfare associated with *X* as: $W(X) = W(U_1(X) \ldots U_N(X))$. The 'aggregating function' *W(X)* is called a *social welfare function (SWF)*. Using the SWF, the socially optimal point for an economy may be identified as that point on an economy's utility possibility frontier which yields the highest level of social welfare (Figure 22, below).

A particular form of the social welfare function is *additive*: $W(X) = \sum_{i=1}^{N} U_i(X)$.

This is sometimes referred to as a *utilitarian* SWF[21]. When the SWF is additive, *X* is socially preferred to *Y* if:

$$W(X) > W(Y) \text{ or } \sum_{i}^{N} U_i(X) > \sum_{i}^{N} U_i(Y) \qquad (55)$$

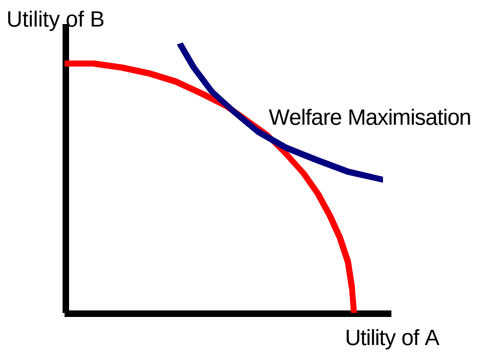

Figure 22
Welfare Maximisation with a SWF

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preferred choice); *independence* (the ranking of two choices should not depend on what the other choices were); *non-dictatorship* (the social ordering should not be imposed).

[21] A generalisation of this form is the weighted sum-of-utilities: $W(X) = \sum a_i U_i(X)$

Inequality and Social Welfare

Suppose there are *N* persons (indexed, *i=1…N*) such that that y_i represents the income of person *i* and that $U(y_i)$ represents the utility associated with his income. Assume that: $U'(y_i) > 0$ and $U''(y_i) < 0$ so that the utility functions are concave functions of income (that is, exhibit strictly diminishing marginal utility). Now suppose that social welfare is additive in the individual utilities:

$$W = \sum_{i=1}^{N} U(y_i) \tag{56}$$

Let **x**={x_i} and **z**={z_i} be two income vectors such that the *Lorenz curve* for **x** lies entirely inside the *Lorenz curve* for **z**. This means that, on the basis of Lorenz-based inequality measures[22], the distribution associated with **z** is more unequal than that associated with **x**.

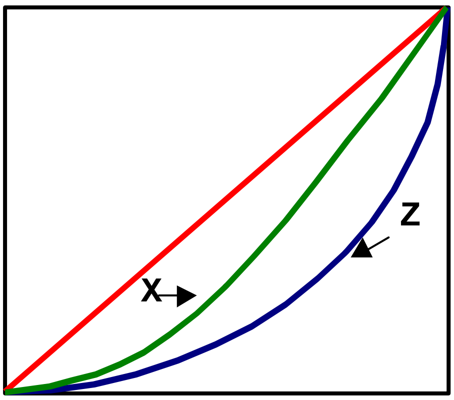

Figure 23: Lorenz-Dominance

Then, by Atkinson's (1970) theorem on Lorenz ranking, *W(**x**)>W(**z**)*. In other words, if one distribution was "more equal" than another, then there would be a higher level of social welfare associated with that distribution; conversely, if for two distributions, **x** and **z**, *W(**x**)>W(**z**)*, then **x** Lorenz-dominates **z** (that is, the Lorenz curve for **x** lies entirely inside the Lorenz curve for **z**: Figure 23).

Let *I(**y**)* be an inequality index, defined over the vector of incomes **y**, which takes values between 0 and1, and which has the property of *mean-independence*. This last property means that the value of the inequality index is unchanged if all incomes are scaled up (or down) by the same factor. Then

[22] For example, the Gini coefficient

if $m(\boldsymbol{y})$ is mean income, the welfare function W of equation (56) may be written as (Sen, 1998):

$$W = m(1 - I) \qquad (57)$$

Equation (57) implies that in evaluating social welfare the contribution of the size of the pie (m) needs to be adjusted downwards by the inequality in its distribution (I). It follows that social welfare could be higher with a lower, than with a higher, mean income, provided that the lower income was sufficiently more equally distributed than the higher income.

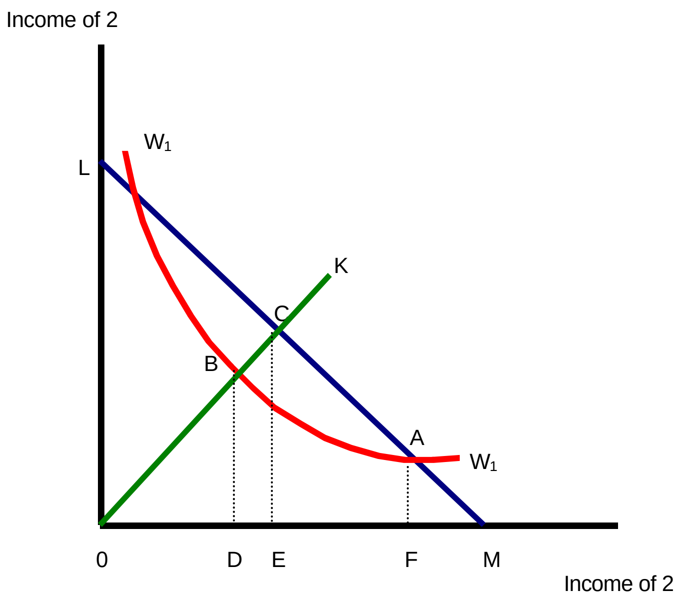

Figure 24
The Equally-Distributed Equivalent Income

These ideas are illustrated in Figure 24, above. The line LM shows the various distributions between 1 and 2 for a given level of income OE. At the point C on LM, both persons get the same income. If the actual distribution is at point A, then the social welfare associated with this is W_1. A lower level of income, OB which is equally distributed between 1 and 2 yields the same level of welfare as the higher level OE distributed according to A. Atkinson (1970) termed OB (<OE) as the "equally distributed equivalent income": *it is the income which, if equally distributed, would be welfare-equivalent to a higher income, distributed unequally*.

The above view of the welfare-reducing effects of inequality raises two questions. First, by how much should welfare be reduced in the face of inequality? Second, is there a link between average income and the degree of inequality in its distribution such that more equality means less income?

Atkinson (1970) showed that the answer to the first question depended on society's "aversion to inequality": the same distribution of income would generate different values of the inequality index, I, in equation (57), depending upon ones aversion to inequality. If society had a high degree of tolerance towards inequality (for example, the USA), the value of the inequality index, and hence the reduction in welfare, would be small; on the other hand, if society was intolerant of inequality (for example, Sweden) the value of the inequality index, and hence the reduction in welfare, would be large.

On the second question, Browning and Johnson (1984) argued that reducing income inequality was not a costless process because the appropriate policies for effecting this reduction produced a misallocation of resources: using a micro dataset for the US, they showed that the marginal cost of reducing inequality could be quite high. Borooah (2002) showed, in the context of a theoretical model, that the equity gains that followed from Fair Employment regulation needed to be offset against the efficiency losses to which such regulation gave rise.

11. Conclusions

This paper focused on five generic causes of market failure:

(i) Imperfection in Competition
(ii) Asymmetry of Information stemming either from 'hidden information' or for from 'hidden action'
(iii) Public Goods
(iv) Externalities
(v) Inequality

However, underlying these generic causes of market failure is a more fundamental question which asks *'what is the market?'*. As Stiglitz (2002) points out, there is not just *one* market model. There are striking differences between say, the Japanese, the Swedish and the US versions of the market economy. The market is at the centre of the Japanese, Swedish and American versions of capitalism but the constellation of factors which buttress the capitalist system is very different in these countries. In Sweden, the government takes on a far more active role in promoting social welfare than in Japan or in the USA; in Japan government and industry work in much closer partnership than in Sweden or in the USA.

So, while both Sweden and Japan depart from the 'competitive version' of market capitalism of the textbook variety – embraced by the USA - they, nevertheless, notch up several *non-market successes*: public health, unemployment benefits and retirement pensions are all far superior in Sweden than in the USA; social cohesion and social order is much greater in Japan than in the USA and this is reflected in very low Japanese crime rates. Nor are standards of living in Japan and Sweden considerably below that in the USA. So while discussing market failure it is also important to bear in mind the notion of non-market success.

As the sections on Public Goods and on Social Welfare pointed out, governments have an important role in providing goods for collective consumption and in promoting social justice. How important this role should be depends on the values that a particular society holds. Much of the 'East Asian miracle' was due to the proactive role of government in: providing universal and high-quality education; supplying the infrastructure – institutional and physical – for growth; and promoting technology in sectors as diverse as agriculture and telecommunications. Much the same point can be made about the role of the government in Ireland in nourishing the 'Celtic tiger'. The economic development of France cannot be explained without an appreciation of the role of the State. So, while advocates of market fundamentalism see governments as part of the problem rather than of the solution, it may be more reasonable to view government – with all its attendant imperfections – as having an important role to play in promoting both economic efficiency and economic equity.

References

Akerlof, G.A. (1970), “The Market for ‘Lemons’: Quality, Uncertainty and the Market Mechanism”, The Quarterly Journal of Economics, vol. 84, pp. 488-500.

Arrow, K.J. (1951), Social Choice and Individual Values, New York: John Wiley.

Atkinson, A.B. (1970), “On the Measurement of Inequality”, Journal of Economic Theory, vol. 2, pp. 244-63.

Baumol, W.J., Panzar, J.C. and Willig, R.D. (1982), Contestable Markets and the Theory of Industrial Structure, New York: Harcourt Brace Jovanovich.

Bergson, A. (1938), "A Reformulation of Certain Aspects of Welfare Economics", Quarterly Journal of Economics, vol. 52, pp. 353-95.

Bergstrom, T., Blume, L. and Varian, H. (1986), “On the Private Provision of Public Goods”, Journal of Public Economics, vol. 29, pp. 25-49.

Black, D. (1948), "On the Rationale of Group decision Making", Journal of Political Economy, vol. 56, pp. 23-34.

Borooah, V.K. (2002), “Employment Inequality, Employment Regulation and Social Welfare”, in A. Levy and J. R. Faria (ed), Economic Growth, Inequality and Migration, Cheltenham: Edward Elgar.

Browning, E.K. and Johnson, W.R. (1984), “The Trade-Off between Equality and Efficiency”, The Journal of Political Economy, vol. 92, pp. 175-203.

Buchanan, J.M. (1965), “An Economic Theory of Clubs”, Economica, vol. 32, pp. 583-99.

Coase, R. (1960), “The Problem of Social Cost”, Journal of Law and Economics, vol. 3, pp. 1-44.

Frantz, R.S. (1988), X-Efficiency: Theory, Evidence and Applications, Boston: Kluwer Academic Publishers.

Harberger, A.C. (1954), “Monopoly and Resource Allocation”, American Economic Review Papers and Proceedings, vol. 44, pp.77-87.

Leibenstein, H.J. (1976), Beyond Economic Man, Cambridge, MA: Harvard University Press.

Liebowitz, S.J. and Margolis, S.E. (1999), Winners, Losers and Microsoft, Oakland, CA: The Independent Insitute.

Lindahl, E. (1919), “Just Taxation – A Positive Solution”, in R. Musgrave and A. Peacock (eds), Classics in the Theory of Public Finance, London: Macmillan.

McGuire, T., Nelson, R. and Spavins, T. (1975), “‘An Evaluation of Consumer Protection Legislation: the 1962 Drug amendments’: a Comment”, Journal of Political Economy, vol. 83, pp. 655-61.

Nelson, P. (1970), “Information and Public Behavior”, Journal of Political Economy, vol. 78, pp. 311-29.

Oi, W. (1973), “The Economics of Product Safety”, Bell Journal of Economics and Management Sciences, vol. 4, pp. 3-28.

Peltzman, S. (1973), "An Evaluation of Consumer Protection Legislation: the 1962 Drug amendments", Journal of Political Economy, vol. 81, pp. 1049-91.

Pindyck, R.S. and Rubinfeld, D.L. (2001), Microeconomics, Upper Saddle River, NJ: Prentice Hall.

Pigou, A.C. (1920), The Economics of Welfare, London: Macmillan

Samuelson, P.A. (1954), "Pure Theory of Public Expenditures", Review of Economics and Statistics, vol. 36, pp. 387-89.

Sen, A.K. (1998), On Economic Inequality, Delhi: Oxford University Press.

Solow, R.M. (1993), "Blame the Foreigner", The New York Review of Books, vol. XL, Number 21, pp. 7-13.

Sorkin, A.L. (1980), The Economics of the Postal Service, Lexington, Mass: Lexington Books.

Spence, M. (1974), Market Signaling, Cambridge, Mass: Harvard University Press.

Stiglitz, J.E. (1989), The Economic Role of the State, Oxford: Basil Blackwell.

Stiglitz, J.E. (2002), Globalisation and its Discontents, London: Allan Lane.

Varian, H.R. (1992), Microeconomic Analysis, New York: W.W. Norton and Company (Third Edition).

Varian, H.R. (2003), Intermediate Microeconomics, New York: W.W. Norton and Company (Sixth Edition).

Weimer, D.L. and Vining, A. (1999), Policy Analysis: Concepts And Practice, Upper Saddle River, NJ: Prentice Hall.

Yellen, J.L. (1984), "Efficiency Wage Models of Unemployment", American Economic Review, vol. 74, pp. 200-205.

Mathematical Appendix

Deriving the Efficiency Conditions

A Pareto efficient allocation is one that maximises one person's utility (A) subject to the constraint that the second person's utility does not change:

$$\underset{X_1^A, X_2^A, X_1^B, X_2^B}{Max} U^A(X_1^A, X_2^A) \text{ s.t. } U^B(X_1^B, X_2^B) = \overline{U}^B \text{ and } T(X_1, X_2) = 0 \quad (58)$$

The Lagrangian for this problem is:

$$L = W[U^A(X_1^A, X_2^A), U^B(X_1^B, X_2^B)] - m[T(X_1, X_2) - 0] \quad (59)$$

and the first order conditions for maximising the Lagrangian with respect to the choice variables are:

$$\frac{\partial L}{\partial X_1^A} = \frac{\partial U^A}{\partial X_1^A} - m\frac{\partial T}{\partial X_1} = 0$$
$$\frac{\partial L}{\partial X_2^A} = \frac{\partial U^A}{\partial X_2^A} - m\frac{\partial T}{\partial X_2} = 0$$
$$\frac{\partial L}{\partial X_1^B} = -l\frac{\partial U^B}{\partial X_1^B} - m\frac{\partial T}{\partial X_1} = 0 \quad (60)$$
$$\frac{\partial L}{\partial X_2^B} = -l\frac{\partial U^B}{\partial X_2^B} - m\frac{\partial T}{\partial X_2} = 0$$

Dividing the first equation by the second, and the third equation by the fourth, yields:

$$MRS_{12}^A = MRS_{12}^B = MRT_{12} \quad (61)$$

Deriving The Samuelson Condition for the Efficient Allocation of a Public Good

$$\underset{X_1, X_2, G}{Max} U_1(X_1, G) \text{ s.t. } U_2(X_2, G) = \overline{U}_2 \text{ and } X_1 + X_2 + C(G) = W_1 + W_2 \quad (62)$$

Form the Lagrangian:

$$L = U_1(X_1, G) - l[U_2(X_2, G) - \overline{U}_2] - m[X_1 + X_2 + C(G) - (W_1 + W_2)] \quad (63)$$

and differentiate with respect to *X_1*, *X_2*, and *G* to obtain the first-order conditions as:

$$\frac{\partial L}{\partial X_1} = \frac{\partial U_1(X_1, G)}{\partial X_1} - m = 0$$
$$\frac{\partial L}{\partial X_2} = -l\frac{\partial U_2(X_2, G)}{\partial X_2} - m = 0 \quad (64)$$
$$\frac{\partial L}{\partial G} = \frac{\partial U_1(X_1, G)}{\partial G} - l\frac{\partial U_2(X_2, G)}{\partial G} - mC'(G) = 0$$

From the first equation: $m = \frac{\partial U_1(X_1, G)}{\partial X_1}$;

from the second equation: $\frac{m}{l} = -\frac{\partial U_2(X_2, G)}{\partial X_2}$;

and from the third equation: $\frac{1}{m}\frac{\partial U_1(X_1, G)}{\partial G} - \frac{l}{m}\frac{\partial U_2(X_2, G)}{\partial G} = C'(G)$

Substituting the appropriate expressions for m and (m/l) into the third equation yields the equilibrium condition:

$$\frac{\partial U_1(X_1, G)/\partial G}{\partial U_1(X_1, G)/\partial X_1} + \frac{\partial U_2(X_2, G)/\partial G}{\partial U_2(X_2, G)/\partial X_2} = C'(G) \quad (65)$$

or, in other words:

$$MRS^1_{XG} + MRS^2_{XG} = MC_G$$

Deriving The Conditions for Welfare Maximisation

The welfare maximisation problem is:

$$\underset{X_1^A, X_2^A, X_1^B, X_2^B}{Max} \; W = W[U^A(X_1^A, X_2^A), U^B(X_1^B, X_2^B)] \text{ s.t. } T(X_1, X_2) = 0 \quad (66)$$

Form the Lagrangian:

$$L = W[U_1(X_1, G), U_2(X_2, G)] - l\,[T(X_1, X_2) - 0] \quad (67)$$

and the first order conditions for maximising the Lagrangian with respect to the choice variables are:

$$\begin{aligned}
\frac{\partial L}{\partial X_1^A} &= \frac{\partial W}{\partial U^A}\frac{\partial U^A}{\partial X_1^A} - l\,\frac{\partial T}{\partial X_1} = 0 \\
\frac{\partial L}{\partial X_2^A} &= \frac{\partial W}{\partial U^A}\frac{\partial U^A}{\partial X_2^A} - l\,\frac{\partial T}{\partial X_2} = 0 \\
\frac{\partial L}{\partial X_1^B} &= \frac{\partial W}{\partial U^B}\frac{\partial U^B}{\partial X_1^B} - l\,\frac{\partial T}{\partial X_1} = 0 \\
\frac{\partial L}{\partial X_2^B} &= \frac{\partial W}{\partial U^B}\frac{\partial U^B}{\partial X_2^B} - l\,\frac{\partial T}{\partial X_2} = 0
\end{aligned} \quad (68)$$

Dividing the first equation by the second, and the third by the fourth, yields the equilibrium conditions: $MRS^A_{12} = MRS^B_{12} = MRT_{12}$.

A Nash Equilibrium Interpretation of Common Property Resources

There are N farmers in a village who graze their cows on the village green. This is owned in common by all the villagers. The number of goats owned by the ith farmer is g_i and $G = \sum_i g_i$ is the total number of goats grazing on the green. The price of a goat is *c* and *v(G)* is the value of the milk from a goat when there are *G* goats grazing.

The maximum number of goats that can be grazed on the green is $\overline{G}$: *v(G)=0* if G= $\overline{G}$, while *v(G)>0* if G< $\overline{G}$

The 'strategy' for each farmer is to choose his g_i from his 'strategy set': *[0, µ]* . The payoff to the farmer

from g_i goats depends upon his choice, as well as upon the choices made by others:

$$p_i = g_i\, v(g_1 + .. + g^*_{i-1} + g_i + g^*_{i+1} + ... + g_N) - cg_i \quad (69)$$

If $g_1^*...g_N^*$ is to be a Nash-equilibrium, then, for each farmer *i, i=1…N,* g_i^* should maximise p_i, given that the other farmers choose $g_j^*, j=1...N, j \neq i$. The first-order conditions for maximising p_i wrt g_i are:

$$v(g_i + \mathbf{g_j^*}) + g_i v'(g_i + \mathbf{g_j^*}) - c = 0 \tag{70}$$

where: $\mathbf{g_j^*} = \sum_{\substack{j=1 \\ j \neq i}}^{N} g_j^*$ and, for a Nash equilibrium, g_i^* solves (1). Consequently, the conditions for a Nash equilibrium are:

$$v(G^*) + g_i^* v'(G^*) - c = 0,\ i = 1...N \tag{71}$$

where: $G^* = \sum_{i=1}^{N} g_i^*,\ v'(G^*) = \dfrac{\partial v(G^*)}{\partial G^*}$

Interpretation: There are G^* goats being grazed, so payoff per goat is $v(G^*)$. A farmer is contemplating adding a goat. This goat will give a payoff of $v(G^*)$ but it will reduce the payoff from his existing goats by the reduction in the payoff-per-goat (after another goat has been added), $v'(G^*)$, times the number of goats he owns, g_i^*. This is his *marginal private benefit* from adding another goat. He compares this marginal private benefit ($v(G^*) + g_i^* v'(G^*)$) to the cost of a goat, *c*, and decides accordingly. Note that when $g_i^* = 1$ (he owns only one goat), $v(G^*) + g_i^* v'(G^*)$ is the new average payoff from a goat.

Summing over the farmers' first-order conditions and dividing by N, yields:

$$v(G^*) + (G^*/N)v'(G^*) - c = 0 \tag{72}$$

and solving (3) yields the Nash equilibrium (total) number of goats, G^*

In contrast, the social optimum number of goats, G^{**} is given by maximising the net revenue to the village from the goats:

$$Gv(G) - Gc \tag{73}$$

and this yields as first-order conditions:

$$v(G^{**}) + G^{**}v'(G^{**}) - c = 0 \tag{74}$$

Interpretation: There are G^{**} goats being grazed, so payoff per goat is $v(G^{**})$. The village is contemplating adding a goat. This goat will give a payoff of $v(G^{**})$ but it will reduce the payoff from the existing goats in the village by the reduction in the payoff-per-goat (after another goat has been added), $v'(G^{**})$, times the number of goats in the village, G^{**}. This is the *marginal social benefit* from adding another goat. The village compares this marginal private benefit ($v(G^{**}) + G^{**}v'(G^{**})$) to the cost of a goat, *c*, and decides accordingly.

Comparing (3) with (5) shows that $G^{*} > G^{**}$. The common resource is over utilised because each villager considers the effect of his action (of grazing another goat) on only his own welfare and neglects the effect of his action upon the other villagers (compare (1) with (5)).