Impure Public Goods

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Club Goods

- A club is a voluntary group of individuals for the shared consumption of one or more goods
- A club good is one which is:
  - jointly consumed by the members of a club
  - not available for consumption to non-members
- A club good is subject to congestion: for a given level of provision, the larger the membership, the less the consumption available per member
- To address the congestion issue, the club operates an exclusion policy

The Club's Decision Problem

The club has two basic decisions to make

- What is the optimal level of provision?
- What is the optimal level of membership?

Mathematical Analysis

- There is a private good (quantity, X) and a club good (quantity, G). The size of the membership is S
- The representative member's utility function is:

$$U = u(X, G, S) \quad \partial U / \partial S < 0, \; S > \bar{S}$$

- The member tries to maximise utility subject to the resource constraint:

$$R = X + [C(G, S) / S]$$

Mathematical Analysis

The cost function is such that:

- Cost increases with the level of provision:

$$(\partial C / \partial G) > 0$$

- Cost increases with the level of membership because of higher maintenance costs:

$$(\partial C / \partial S) > 0$$

Mathematical Analysis

Differentiating the Lagrangian function:

$$L = U(X, G, S) + \lambda[R - C(G, S)/S]$$

Yields the first order conditions:

$$MRS_{XG} = \frac{\partial U / \partial G}{\partial U / \partial X} = \frac{\partial C}{\partial G} \times \frac{1}{S} \text{ (provision)}$$

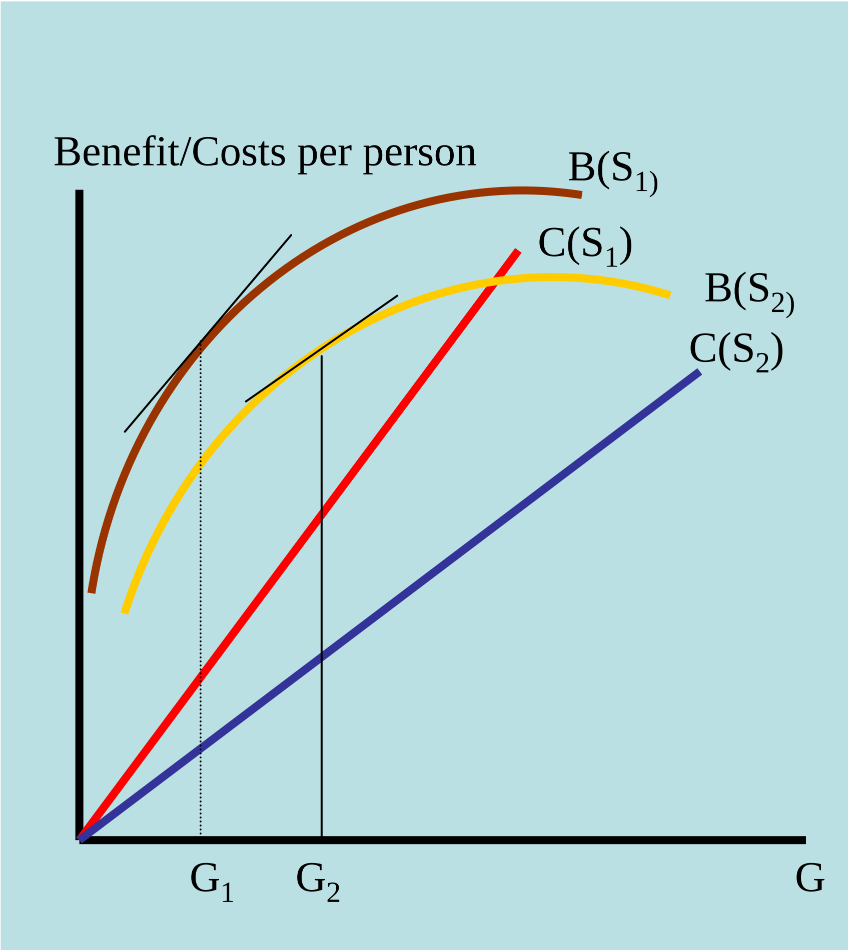
$$MRS_{XS} = \frac{\partial U / \partial S}{\partial U / \partial X} = \frac{1}{S}\frac{\partial C}{S} - \frac{C(G, S)}{S^2} \text{ (membership)}$$

Mathematical Analysis

- The *provision* condition says that the MRS between the private and club good is equal to the member's share of the marginal cost of provision of the public good
- The membership condition says that MRS between the private good and membership is equal to the marginal cost of increasing membership:
  - Increased membership increases provision cost
  - Increased membership reduces a member's cost
- Cross-multiplying the provision condition by S yields the Samuelson condition:

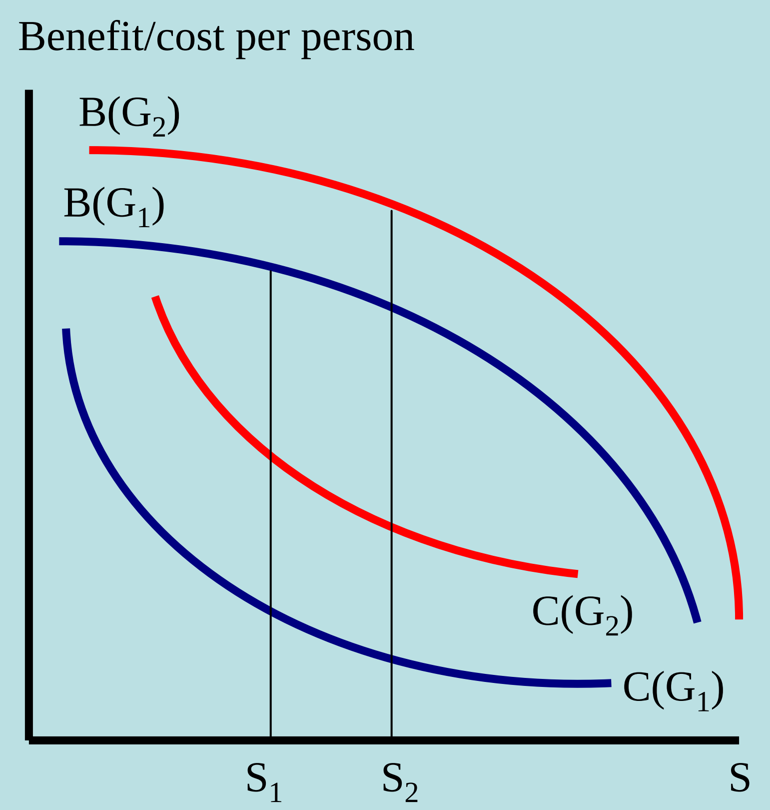
$$\sum MB_G = MC_G$$

The Optimal Provision of a Club Good



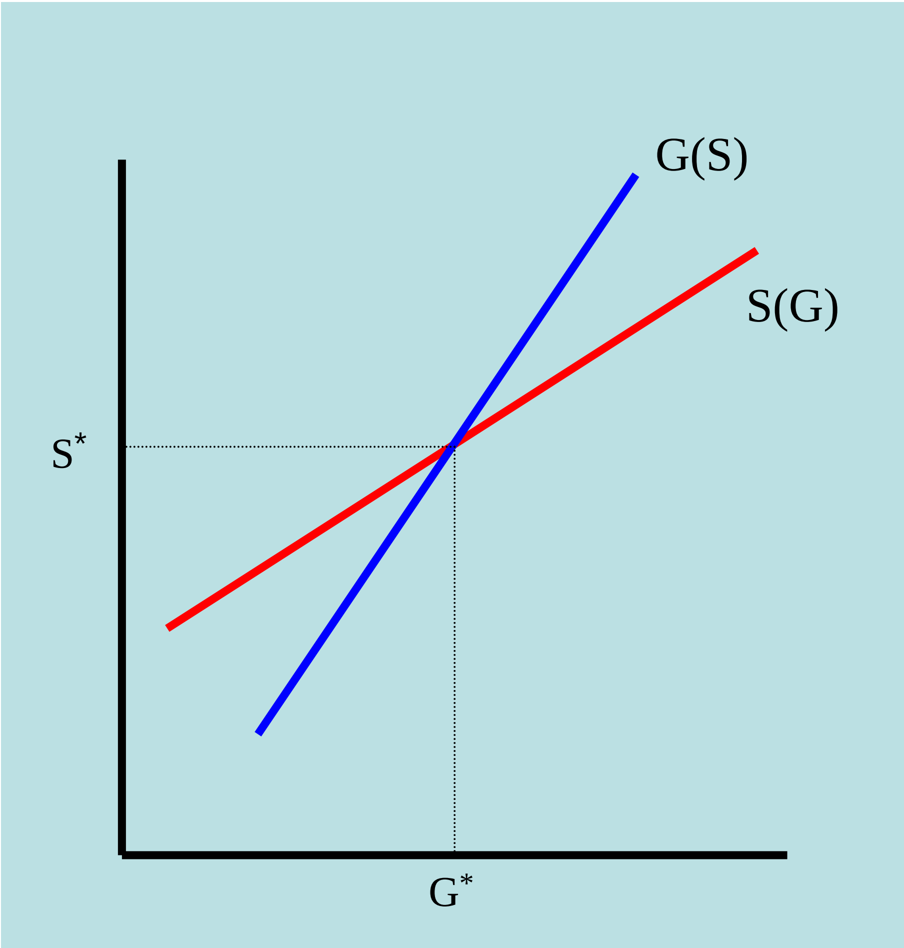
- The club good is produced under constant returns
- $C(S_1)$ and $C(S_2)$ are the costs curves for different membership levels: $S_1 < S_2$
- $B(S_1)$ and $B(S_2)$ are the corresponding benefit curves
- G_1 and G_2 are optimal provisions at S_1 and S_2

The Optimal Level of Membership



- The curves $B(G_1)$ and $B(G_2)$ show the benefits, for G_1 and G_2 levels of provision, for different levels of membership
- The curves $C(G_1)$ and $C(G_2)$ show the costs, for G_1 and G_2 levels of provision, for different levels of membership
- The optimal levels of membership, S_1 and S_2, maximise the distance between the benefit and cost curves

Optimal Level of Provision and Membership



- G(S) shows the optimal level of provision for different levels of membership
- S(G) shows the optimal membership level for different levels of provision
- The equilibrium level of provision and membership are given at the point of intersection
- For stability, the S(G) curve should be flatter than the G(S) curve

Community Size and the Tiebout Hypothesis

- Suppose that the clubs represent local communities
- The club good is a package of health, education, transport etc. supplied to the local population
- For each community there is an optimal population, S^*, which will maximise the net benefit from a package
- If $S > S^*$, people will leave the community
- If $S < S^*$, people will enter the community
- People will “vote with their feet” (Tiebout)

Fishing Example

Fishermen	Total	Average	Marginal
1	10	10	10
2	18	9	8
3	24	8	6
4	28	7	4
5	30	6	2
6	30	5	0

Tragedy of the Commons

- Suppose the price of fish is £1 per fish and the outside wage is £6
- Then 5 fishermen will use the lake: outside wage = average product
- But, from society's perspective, only 3 fishermen should use the lake: fishermen 4 and 5 would be better employed outside the fishing industry
- http://en.wikipedia.org/wiki/Tragedy_of_the_commons
- http://dieoff.org/page95.htm

Congestion Without Exclusion

- There are N profit maximising firms, indexed $i=1\ldots N$, each firm having access to a fishing ground
- Each firm has a fishing fleet of size R_i where: $R=\Sigma R_i$ is the size of the industry fleet
- The total catch of the industry is: $Y=f(R)$
- The total catch of each firm is:

$$Y_i=(R_i/R)f(R)=(R_i/[R_i + \bar{R}])f(R_i + \bar{R})$$

where: $\bar{R}=R-R_i$ is the “other firms” aggregate fleet

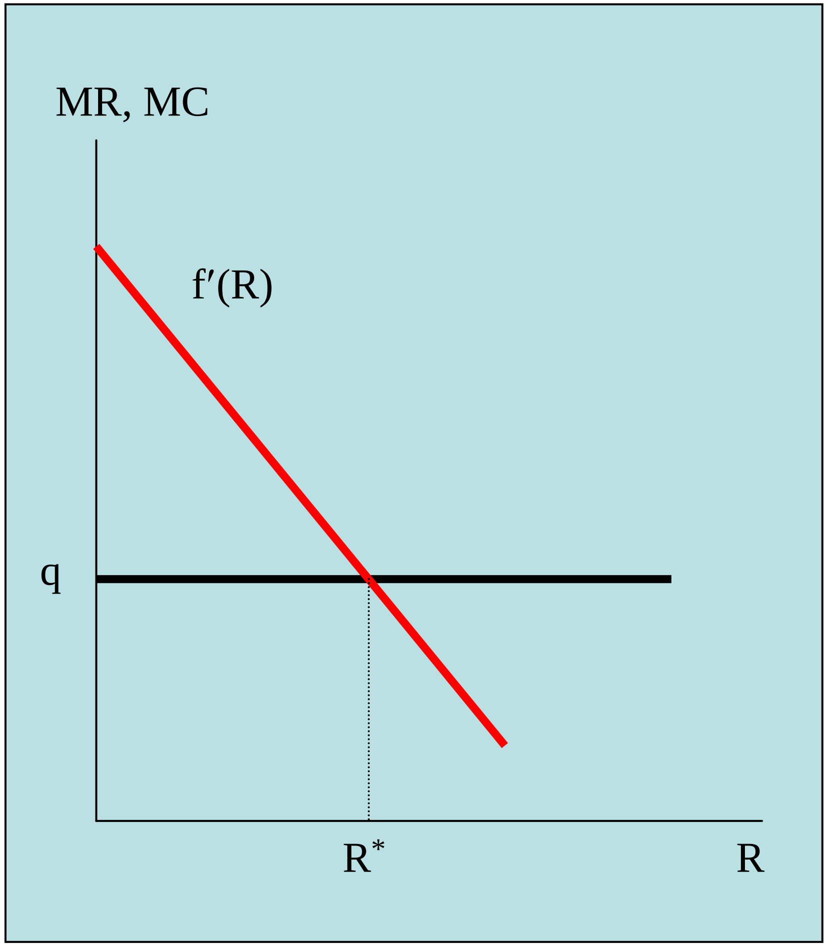
The Industry

- The industry will maximise profits by choosing the fleet size, R, to maximise:

  $$\Pi = p \times f(R) - q \times R$$

  where: p is the price of fish and q is the price of a fishing boat
- Setting $p=1$, the optimal value, R^*, is given by the condition: $f'(R) = q$

Industry Equilbrium



R^* is the size of industry's profit maximising fleet

$Y^*=f(R^*)$ is the optimal catch

$\Pi^*=Y^*- qR^* =f(R^*)-qR^*$ is maximum profits

The Firm

- Each firm will choose R_i to maximise its profits:

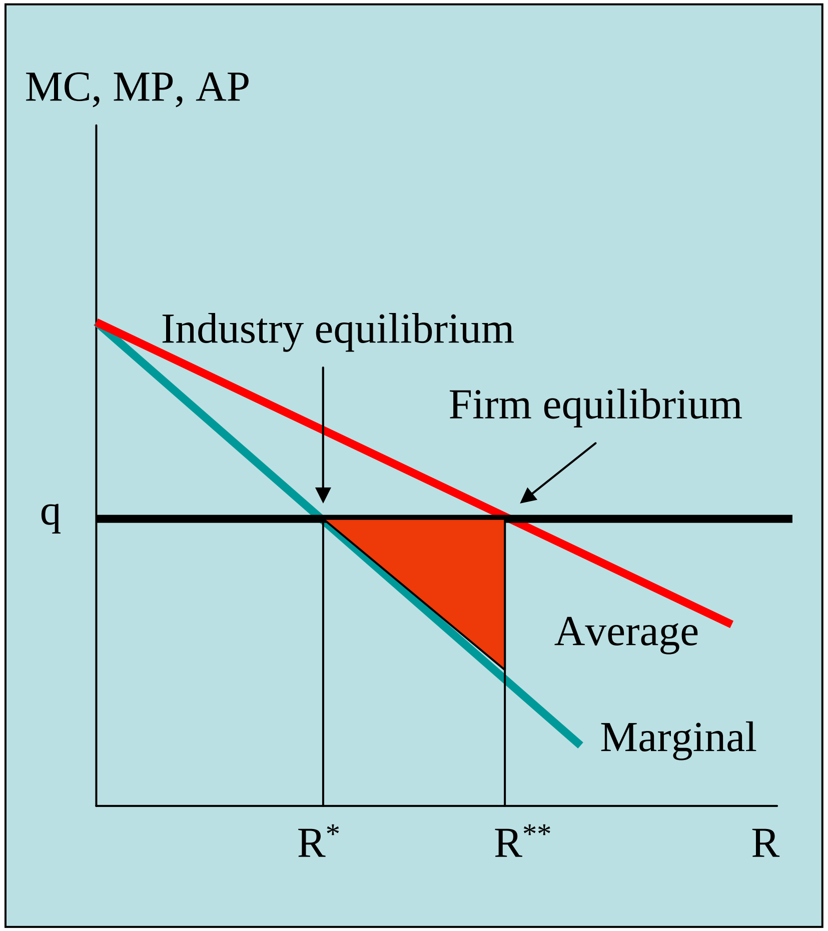
$$\Pi_i = p(R_i/[R_i + \bar{R}])f(R_i + \bar{R}) - qR_i$$

- Differentiating the profit function wrt R_i and (assuming the firms are all of equal size) setting to zero yields:

$$q = (\bar{R}/R) \times (f(R)/R) + (R_i/R)f'(R)$$

$$= \frac{N-1}{N} \times \frac{f(R)}{R} + \frac{1}{N} \times f'(R)$$

Firm Equilibrium

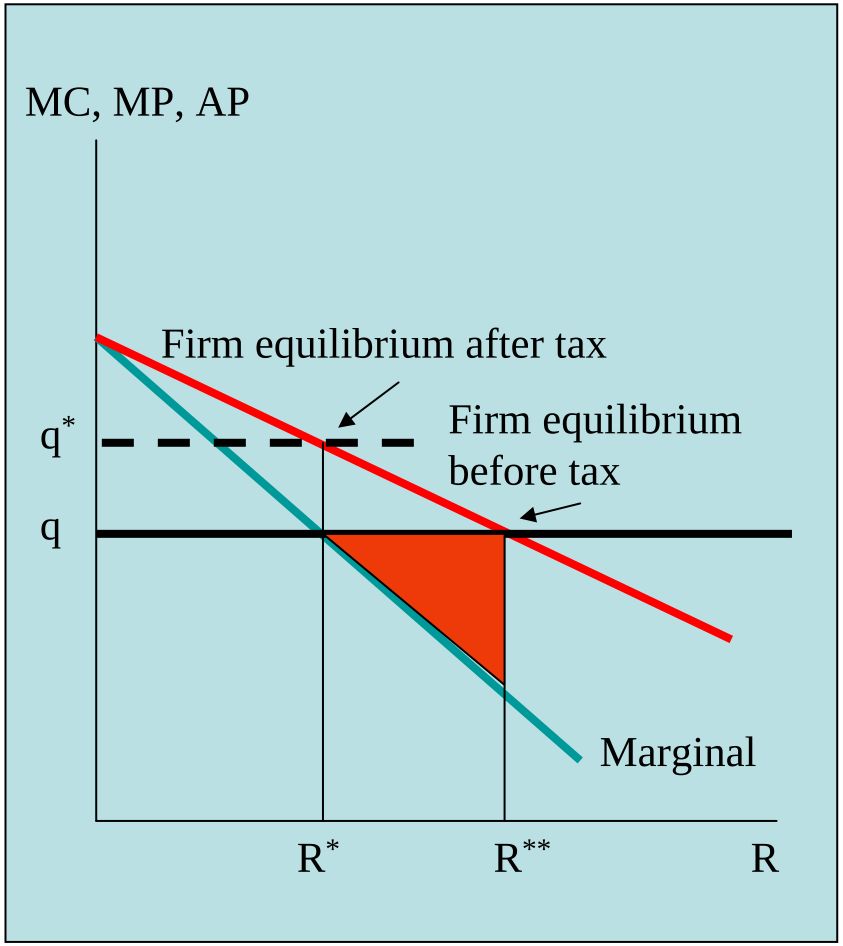


- The firm in equilibrium equates *q* to a *weighted average* of average product and marginal product
- When the number of firms N is very large ($N \to \infty$): q = average product
- When the marginal falls, the average falls
- When the average is falling, the marginal lies below the average
- So, over fishing will result: the red triangle measures the loss to society from over fishing

Property Rights

- Over fishing arises because of an absence of property rights
- Since no one owns the fishing ground – it is a *common property resource* – firms can fish without cost
- One solution is to assign property rights and to impose a charge per fishing boat

Firm Equilibrium with Tax



- A tax per fishing boat is imposed: $q^* - q = AP(R^*) - MP(R^*)$
- This raises the cost per boat from q to q^*
- When firms are in equilibrium, there will a total of R^* boats

Appendix

1. $$\frac{\partial \Pi_i}{\partial R_i} = \frac{(R_i + \bar{R}) - R_i}{(R_i + \bar{R})^2} \times f(R) + \frac{R_i}{(R_i + \bar{R})} \times f'(R) - q$$

$$= \frac{\bar{R}}{R} \times \frac{f(R)}{R} + \frac{R_i}{R} \times f'(R) - q = 0$$

2. $$d\left(\frac{f(R)}{R}\right) / dR < 0 \Rightarrow \frac{f'(R) \times R - f(R)}{R^2} < 0$$

$$\Rightarrow f'(R) < \frac{f(R)}{R}$$