Game Theory and Strategic Behaviour

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The Prisoners' Dilemma

Prisoner Y

Prisoner X

	Confess	Deny
Confess	-5,-5	0,-10
Deny	-10,0	-1,-1

Elements of a Game:

Players: agents participating in the game (X and Y))

Strategy Set: Actions that each player may take under any possible circumstance (Confess, Deny)

Strategy: An action that a player takes

Outcomes: The various possible results of the game (four, each represented by one cell of matrix)

Payoffs: The cost/benefit that each player gets from each possible outcome of the game (the prison sentences entered in each cell of the matrix)

Points to note

1. The Payoff to a player depends not just on his chosen action but also on the action chosen by the other player

- So, there is ***inter-dependence***

2. Each player chooses his action, without knowing the other's choice

- It is a **non-co-operative game**

3. It is played once: it is a "**one-shot**" game

A Solution to a Game

- Each player chooses a strategy that gives him the highest payoff, given the choices of others
- If no player wishes to change his strategy, given the choice of the others, the game has a *solution* and the players arrive at an *equilibrium*
- The solution is the set of strategies chosen by the players
- Question is: will a game have a solution?

Dominant Strategy

- A dominating strategy is one which gives the player's his highest payoff, ***regardless*** of the other players choice of action

➢ In the PD game, confess is the dominating strategy for X:

1. If Y denies, X is better off confessing (0 years)
2. If Y confesses, X is better off confessing (5 years)

➢ Confess is also the dominating strategy for Y

Solution to the game is for both to confess and spend 5 years in jail

Collective versus self-interest

- For both to confess (5 years in jail for each) is not the optimal strategy
- The optimal strategy (1 year in jail for each) is for both to deny
- But the optimal strategy is inaccessible, because each cannot be sure what the other will choose: **there is an absence of trust!**

Building a New Plant

Ford

General Motors

	Build	Don't Build
Build	16, 16	20, 15
Don't Build	15, 20	18, 18

Solution to the Ford-GM Game

- The strategy “build” dominates the strategy “don’t build”
- But the optimal strategy is “don’t build”
- Think of India and Pakistan building a nuclear bomb
- Same conclusion: “build a bomb” is the dominant strategy, but “don’t build” is the best strategy
- But absence of trust means that “best strategy” is not chosen

Dominated Strategy

- The opposite of a “dominant” strategy is a “dominated” strategy
- A dominated strategy will always give a player a lower payoff than the alternative strategies, regardless of the strategy choice of the other players
- A rational player will never play a dominated strategy

Honda \ Toyota	Builld	Do not Build
Build	12,4	20,3
Do not build	15,6	18,5

Elimination of Dominated strategies

In the game alongside, Honda does not have a dominant strategy: if Toyota builds, it is better for Honda to not build; if Toyta does not build, it is better for Honda to build

But Toyata has a dominant strategy: whether Honda builds or not, it is better for Toyota to build

So, Toyota will build and Honda will not build: SOLUTION!

1. Both players do not need a dominant strategy. If one player has a dominant strategy, its other (dominated) strategy can be eliminated
2. Game theory teaches us to put ourselves in the mind of our rival – Honda argues that Toyota will build and so Honda should not build

		Toyota	
	Build large	Build small	Do not build
Build large	0,0	12,8	18,9
Build small	8,12	16,16	20,15
Do not build	9,18	15,20	18,18

Honda

Neither Toyota nor Honda has a dominant strategy

But, for Toyota, “build large” is dominated by other strategies: Toyota will never choose “build large”

For Honda, “build large” is dominated by other strategies: Honda will never choose “build large”

So “build large” can be eliminated for both

Elimination of Dominated Strategies

		Toyota	
		Build Small	Do not Build
Honda	Build Small	16,16	20,15
	Do not build	15,20	18,18

Elimination of Dominated strategies

In the game alongside, Honda does have a dominant strategy: if Toyota builds small, it is better for Honda to build small; if Toyota does not build, it is better for Honda to build small

Toyota has a dominant strategy: whether Honda builds small or does not build, it is better for Toyota to build small

So, Toyota and Honda will both build small: SOLUTION!

1. Both players do not need a dominant strategy to start with.
2. However, after eliminating their dominated strategies, they do have dominant strategies

Obtaining a Solution to a Game

- If both players have a ***dominant*** strategy, they will choose it: Prisoner's Dilemma
- If one player has a ***dominant*** strategy, he will choose it, the other player's strategy will be his best response to this (Toyota-Honda)
- If neither player has a dominant strategy, successively eliminate each player's dominated strategies to simplify the game and look for ***dominant*** strategies

Nash Equilibrium

- A dominant strategy for a player has to be his best strategy for ***every*** choice of strategy by the rival
- This is a very demanding requirement and very often it will not be met, even after eliminating dominated strategies
- A weaker requirement is that a player's choice is best ***for the best choice of his rival***
- A pair of strategies is a ***Nash equilibrium*** if player 1's choice is optimal, given 2's optimal choice and player 2's choice is optimal, given 1's optimal choice

Finding a Nash Equilibrium

- If a dominant can't be found, identify each player's best strategy, given the strategy of the other player
- The combination of best strategies is the ***Nash equilibrium***

Neither player has a dominating nor a dominated strategy

For each player we identify his best strategy, given the other player's strategy: red for player 1, green for player 2

Two equilibrium points: (A,E) or (C,D)

Player 1	Player 2	Strategy D	Strategy E	Strategy F
	Strategy A	4,2	13,6	1,3
	Strategy B	3,10	0,0	15,2
	Strategy C	12,14	4,11	5,4

Neither player has a dominating nor a dominated strategy

For each player we identify his best strategy, given the other player's strategy: red for player 1, green for player 2

Two equilibrium points: (Top, Left) or (Bottom, Right)

		Player 2: Left	Player 2: Right
Player 1	Top	2,1	0,0
Player 1	Bottom	0,0	1,2

Cournot Duopoly

There are two firms: y_1 and y_2 are their respective outputs

$Y=y_1+y_2$ is industry output: price (p) depends on Y

The payoffs are firm profits:

$\pi_1=p(Y)y_1-C(y_1)=p(y_1+y_2)y_1 - C(y_1) = \pi_1(y_1, y_2)$

$\pi_2=p(Y)y_2-C(y_2)=p(y_1+y_2)y_2 - C(y_2) = \pi_2(y_1, y_2)$

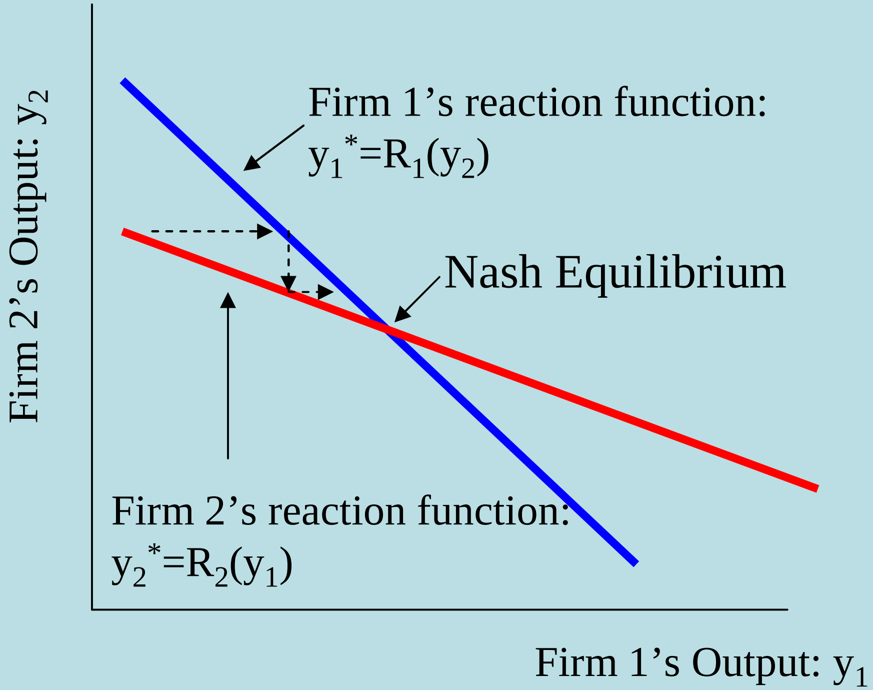
The payoffs depend upon own output and output of rival

Firm 1 chooses y_1 to maximise π_1, given y_2: $y_1^*= R_1(y_2)$

Firm 2 chooses y_2 to maximise π_2, given y_1 :$y_2^*= R_2(y_1)$

Cournot-Nash Equilibrium

Two Firms



- The dashed arrows represent the path towards the Cournot-Nash equilibrium
- At the Cournot-Nash equilibrium, no firm has an incentive to change

Betrand Duopoly

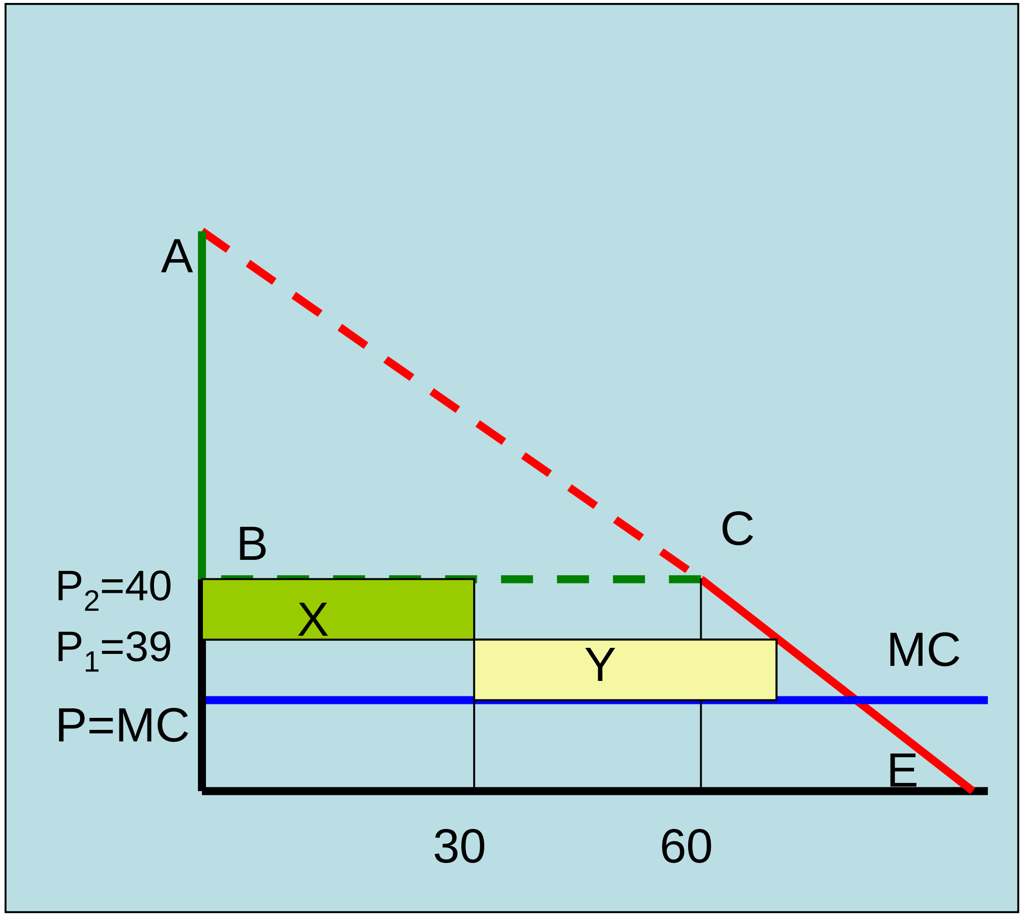
- In Cournot-Nash firms compete on output
- In a Betrand model, firms compete on price

  There are two firms, producing a homogenous product with marginal cost (MC)

  They price this at p_1 and p_2, respectively

  D(p) is the aggregate demand curve

Marginal Cost Pricing Under Betrand Equilibrium



If p_2=40, firm 1 will sell nothing if $p_1 > p_2$ and will capture the market if $p_1 < p_2$

So firm 1's residual demand curve is ABCD

If firm 1 undercuts to $39, its loss is X but its gain is Y

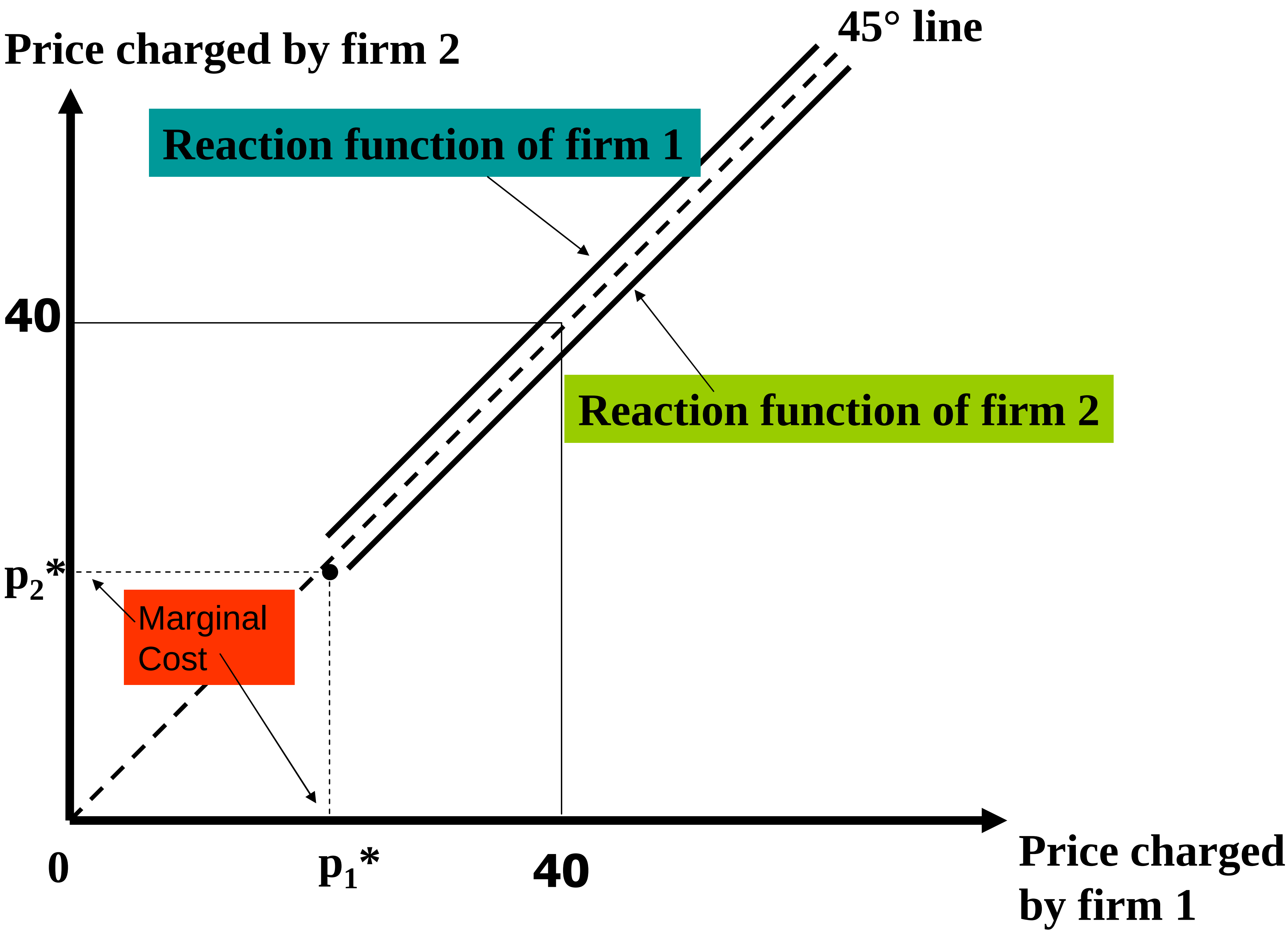
So, provided Y>X, it will undercut

Then, firm 2 will undercut 1, by reducing price below $39

Process continues until $p_1 = p_2 = MC$

Unprofitable to lower price below MC

Reaction Functions Under Betrand Duopoly with Homogenous Product



Betrand Duopoly and Competitive Equilibrium

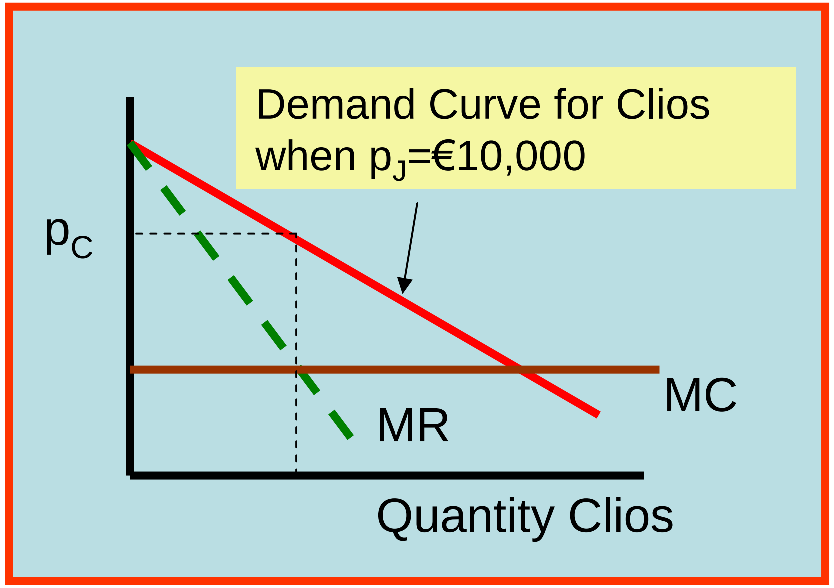
1. Firms price at marginal cost

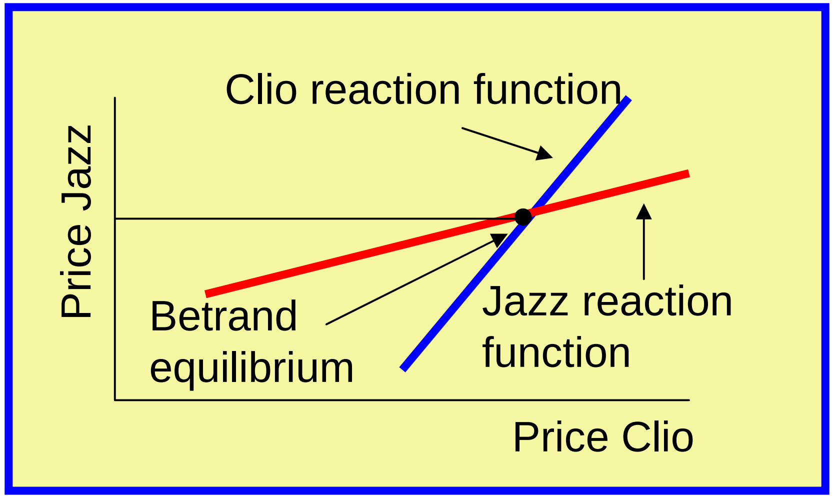
2. Firms make zero profits

3. The number of firms is irrelevant to the price level as long as more than one firm is present: two firms are enough to replicate the perfectly competitive outcome!

…essentially, the assumption of no capacity constraints combined with a constant average and marginal cost takes the place of free entry…

Betrand Duopoly with Differentiated Products





There are two products: Honda's Jazz and Renualt's Clio. For each product, demand depends on own price and rival's price:

$D_{Clio}=f(p_C, p_J)$ and $D_{Jazz}=g(p_C, p_J)$

For each price of Jazz, Clio will have a profit maximising price, p_C

We can compute, for every Jazz price p_J, the corresponding profit maximising Clio price, p_C: this is the Clio reaction function

We can compute, for every Clio price p_C, the corresponding profit maximising Jazz price, p_J: this is the Jazz reaction function

Betrand equilibrium is at intersection of reaction functions

Neither player has a dominating nor a dominated strategy

For each player we identify his best strategy, given the other player's strategy: red for player 1, green for player 2

Player 2

Player 1		Left	Right
	Top	0,0	0,-1
	Bottom	1,0	-1,3

There is no Nash equilibrium

Mixed Strategies

- Suppose A plays top with probability $\frac{3}{4}$ and bottom with probability $\frac{1}{4}$
- Suppose B plays left and right with probabilities $\frac{1}{2}$
- Then p(top, left)=p(top,right)=3/8
- And   p(bottom, left)=p(bottom,right)=1/8
- This yields a Nash equilibrium